(1)

27.10.2020

• použili sme $w(E) \sim \sqrt{E}$ (osmina gule v (n_x, n_y, n_z) priestore)

$E = \frac{p^2}{2m}$

inak

$$\sum_i \rightarrow \int \frac{d^3x\, d^3p}{(2\pi\hbar)^3} = \frac{V g_s}{(2\pi\hbar)^3} \int_0^\infty dp\, p^2\, 4\pi = \frac{g_s V}{(2\pi\hbar)^3} \frac{(2m)^{3/2}}{2} 4\pi \int_0^\infty dE \sqrt{E} =: \int_0^\infty dE\, w(E)$$

$p^2 \rightarrow 2mE$

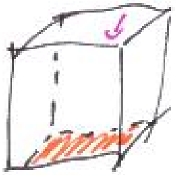
$dp \rightarrow \sqrt{2m}\, \frac{1}{2} \frac{1}{\sqrt{E}} dE$

definícia $w(E)$

$$= \frac{V g_s}{4\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^\infty dE \sqrt{E}$$

POZOR NA FAKTOR ZA SPINOVÚ DEGENERÁCIU

častice



– koľko bozónov sa usadí závisí od teploty a pri tepl. T_c je menej ako dané N

– pre $T < T_c$ je v kondenzáte $N\left(1 - \left(\frac{T}{T_c}\right)^{\frac{3}{2}}\right)$ častíc

• AKO SA TO PREJAVÍ NA FYZIKÁLNYCH VLASTNOSTIACH? → tepelná kapacita

↳ pre $T < T_c$, $\mu = 0$

$$\langle E \rangle = \int_0^\infty dE \frac{w(E)}{e^{\beta E} - 1} E = \frac{V (m k T)^{5/2}}{\sqrt{2}\, \pi\, \hbar^3} \int_0^\infty dx \frac{x^{3/2}}{e^x - 1} = \#\, T^{5/2}$$

$x = \beta E$

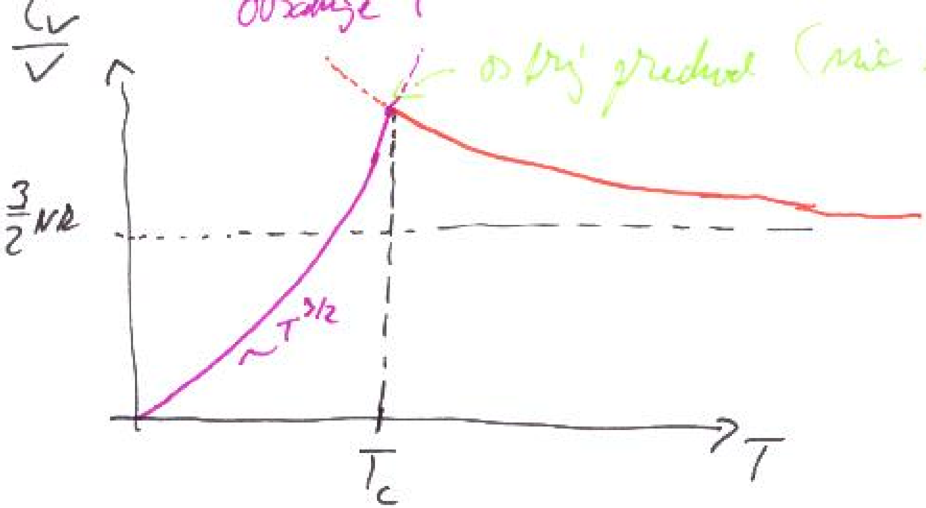
tepelná kapacita na jednotkový objem $\frac{C_V}{V} = \frac{1}{V} \frac{\partial \langle E \rangle}{\partial T} \sim T^{3/2}$

Teraz pre $T > T_c$ $\quad \langle E \rangle = \frac{V(mkT)^{3/2}}{\sqrt{2}\pi\hbar^3} \int_0^\infty dx \frac{x^{3/2}}{e^x e^{-\beta\mu} - 1}$

potom $\quad \frac{C_V}{V} = \frac{1}{V}\frac{\partial\langle E\rangle}{\partial T} = \# \; T^{3/2} + \frac{V(mkT)^{3/2}}{\sqrt{2}\pi\hbar^3}\left(\frac{\partial\int_0^\infty \dots}{\partial e^{-\beta\mu}}\right)\left(\frac{\partial e^{-\beta\mu}}{\partial T}\right)$

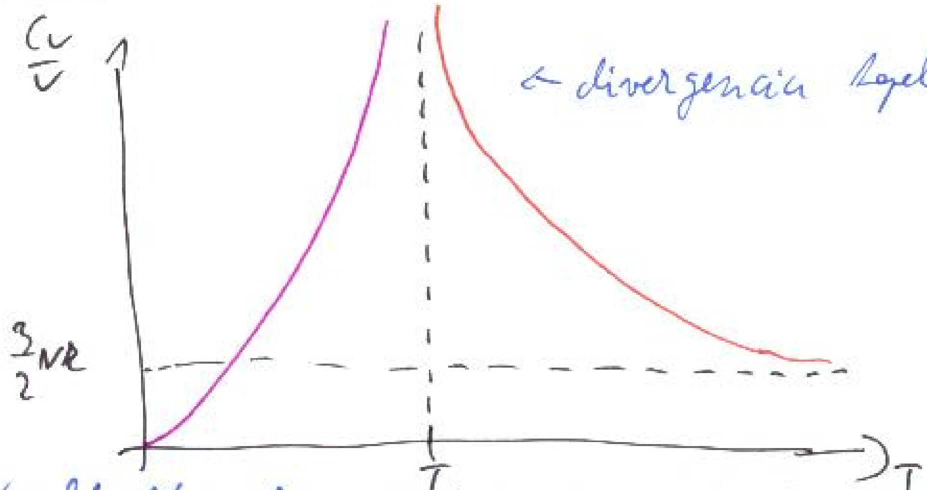
↑ obsahuje T

výsledok



← pre ideálny bozónový plyn

s interakciou



← divergencia tepelnej kapacity

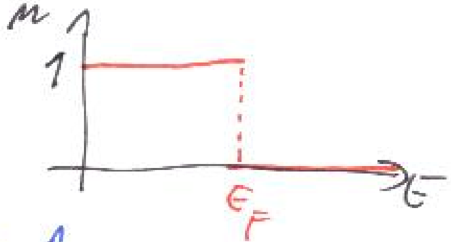
$dE = C_V \cdot dT$

napr. latentné teplo topenia ľadu

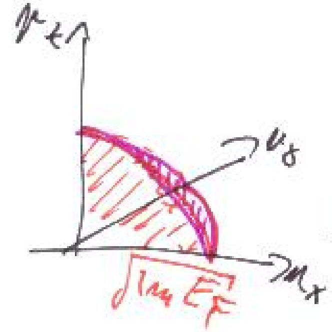
- striktné nespojitosti / nehladký prechod dostávame iba v limite $N \to \infty$ (nespojitosti sú stále $\frac{1}{N}$)

PARAMAGNETIZMUS

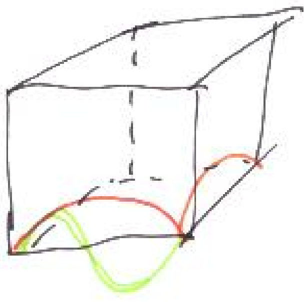
↳ v F.D. rozdelenia some dostali E_F



⟹



prístav fyzikálny



n_x	n_y	n_z	spin $\frac{1}{2}$
1	0	0	↑
1	0	0	↓
0	1	0	↑
0	1	0	↓
0	0	1	↑
0	0	1	↓
1	1	0	↑
1	1	0	↓
1	0	1	↑

} rovnaká energia

Bohrov magnetón ↓

↳ do vonkajšieho magnetického poľa B ⟹ spin ↑ energia $\mu_B B$, spin ↓ energia $-\mu_B B$

$$\mu_B = \frac{|e|\hbar}{2mc}$$

F.D. rozdelenie

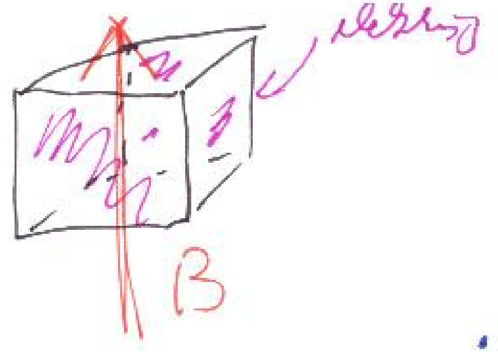
$$\langle n_k \rangle \sim \frac{1}{e^{\beta(E \pm \mu_B B - \mu)} + 1} \Rightarrow$$

$$N_\uparrow = \frac{g V}{4\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2} \int_0^\infty dE \frac{\sqrt{E}}{e^{\beta(E + \mu_B B - \mu)} + 1}$$

(g = 1 lebo iba spin hore)

$$N_\downarrow = \frac{g V}{4\pi^2}\left(\frac{2m}{\hbar}\right)^{3/2} \int_0^\infty dE \frac{\sqrt{E}}{e^{\beta(E - \mu_B B - \mu)} + 1}$$

(g = 1 lebo iba spin dole)



(4.

Magnetizácia $M=\mu_B\,(N_\uparrow - N_\downarrow)$ — pretože spin naopak dáva kladný magnetický moment

susceptibilita $\chi = \dfrac{\partial M}{\partial B}$ (niečo ako tepelná kapacita)

• ako vyzerá χ pri veľkej a pri malej teplote?

↳ pre veľké teploty:

$$\int_0^\infty dE\,\frac{\sqrt{E}}{e^{\beta(E\pm\mu_B B-\mu)}+1} = \int_0^\infty dE\,\frac{e^{-\beta(\dots)}\sqrt{E}}{1+e^{-\beta(\dots)}} = \int_0^\infty dE\,\sqrt{E}\,e^{-\beta(\dots)}\big(1-\underbrace{e^{-\beta(\dots)}+\dots}_{\approx 0}\big) =$$

pre veľké teploty $e^{\mu\beta}\to 0$ alebo $\mu\to-\infty$ kvôli fixovaniu N

$$= \int_0^\infty dE\,\sqrt{E}\,e^{-\beta(E\pm\mu_B B)}\,e^{\beta\mu} = e^{\beta\mu\mp\mu_B B\beta}\int_0^\infty dE\,\sqrt{E}\,e^{-\beta E} = \frac{e^{\beta\mu\mp\mu_B B\beta}}{\beta^{3/2}}\,\Gamma\!\left(\tfrac{3}{2}\right)$$

$$= \frac{e^{\beta\mu\mp\mu_B B\beta}}{\beta^{3/2}}\,\frac{\sqrt{\pi}}{2}$$

pre magnetizáciu $M=\mu_B(N_\uparrow-N_\downarrow)=\dfrac{\mu_B V}{\lambda^3}\,e^{\beta\mu}\left[e^{+\mu_B B\beta}-e^{-\mu_B B\beta}\right]=\dfrac{\mu_B V}{\lambda^3}\,2\sinh\mu_B B\beta$

$$\lambda = \sqrt{\frac{2\pi\hbar^2}{mkT}}$$

$$= \frac{\mu_B V}{\lambda^3}\,e^{\beta\mu}\big(2\sinh(\mu_B B\beta)\big)$$

μ určíme z celkového počtu častíc: $N=N_\uparrow+N_\downarrow$

$$N = \frac{V}{\lambda^3} e^{\beta\mu} \left(2\cosh \beta\sigma\mu_B B \right) \Rightarrow \boxed{M = \sigma\mu_B N \tanh(\beta\sigma\mu_B B)}$$

$N \uparrow$ $N_\uparrow + N_\downarrow$

to isté ako mikrokanonická rekurzia spinov

pre susceptibilitu dostaneme Curieho zákon

$$\boxed{\chi(B=0) = \frac{N\sigma\mu_B^2}{kT} \sim \frac{1}{T}}$$

↳ pre nízke teploty:
$$\int_0^\infty dE \frac{\sqrt{E}}{e^{\beta(E \pm \sigma\mu_B B - \mu)} + 1} \xrightarrow[T \to 0]{F.D.} \int_0^{E_F \mp \sigma\mu_B B} dE \sqrt{E} = \frac{2}{3} E^{3/2} \Big|_0^{E_F \mp \sigma\mu_B B}$$

magnetizácia
$$M = \sigma\mu_B (N_\uparrow - N_\downarrow) = \frac{\sigma\mu_B V}{6\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \left[(E_F - \sigma\mu_B B)^{3/2} - (E_F + \sigma\mu_B B)^{3/2} \right] =$$

$$= \# \cdot E_F^{3/2} \left[\left(1 - \frac{\sigma\mu_B B}{E_F}\right)^{3/2} - \left(1 + \frac{\sigma\mu_B B}{E_F}\right)^{3/2} \right] = -\# \cdot E_F^{1/2}\, 3\sigma\mu_B B$$

$\left(1 - \frac{3}{2}\frac{\sigma\mu_B B}{E_F}\right)$ $\left(1 + \frac{3}{2}\frac{\sigma\mu_B B}{E_F}\right)$

degenerácia fermiho energie $\omega(E_F)$ aj so spinovou degeneráciou

$$= -\sigma\mu_B^2 \frac{g_s V}{2 \cdot 2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} E_F^{1/2} B \Rightarrow \boxed{\chi = \sigma\mu_B^2\, \omega(E_F)}$$

počet elektrónov s energiou rovnou E_F

POZOR NA FAKTORY 2, DOPLNIŤ $g_s = 2$ PRE CELKOVÚ DEGENERÁCIU OBOCH SPINOV

KLASICKÁ LIMITA F.D. & B.E.

↳ stredná obsadenosť jednočasticového stavu je veľmi malá ⟺ pri vysokej teplote

$$\langle n_r \rangle = \frac{1}{e^{\beta(E-\mu)} \pm 1} \ll 1$$

± 1 ↓ nie je podstatné

$$\langle n_r \rangle \to e^{\beta(\mu - E)}$$

↓ v klasickej limite nie je rozdiel medzi bozónmi a fermiónmi

pre celkový počet častíc

$$\langle N \rangle = \int dE \, \omega(E) \, e^{\beta(\mu - E)} = \boxed{\frac{V}{\lambda^3} e^{\beta\mu}}$$

← to čo sme už raz dostali pre klasický plyn v GC rozdelení