Introduction to string theory
Homework 1

Feel free to direct any questions to
`juraj(a)tekel(b)gmail(c)com`

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Príklad 1. Show that from the Lagrangian in the action for a point particle

$$S = -m \int dt \sqrt{1 - \frac{d\vec{x}}{dt} \cdot \frac{d\vec{x}}{dt}}$$

we get the expected form of momentum and energy.

Find the canonically conjugated momenta to the variables X^μ in the action

$$S = -m \int d\tau \sqrt{-\dot{X} \cdot \dot{X}}$$

and show that all 4 are not independent.

Príklad 2. Let us have an action for the fields $X^\mu(\tau)$ and $e(\tau)$

$$S = -\frac{1}{2} \int d\tau \left(e^{-1} \dot{X} \cdot \dot{X} - e m^2 \right) \ .$$

Derive the equation of motion for the field e. Then show that using this equation, the action changes to the original version of the action for a point relativistic particle in square root form from the previous problem. Also show that equation of "motion" for e is equivalent to the canonical momenta constraint from the same problem.

Príklad 3. Let us have the Nambu-Goto action

$$S = -T \int d^2\sigma \sqrt{-(\dot{X} \cdot \dot{X})(X' \cdot X') + (\dot{X} X')^2} \ .$$

Write this action in coordinates on the worldsheet (in gauge), in which $\tau = t/R$ and in the system[1], where the instantaneous velocity of the string is $\frac{d\vec{x}}{dt} = 0$. Show that from this form we can conclude that the potential energy of the string is

$$T \times \text{length of the string} \ .$$

Show that in units of $c = \hbar = 1$ the dimension of the tension is $[m]^2$ and hence we expect that

$$T \sim \frac{1}{l_p^2} \ .$$

Príklad 4 (Stringo)**.** Go to archive of presentations from the Strings 2024 conference and watch one of your choice. Write down words, concepts, terms, and similar things that you have never heard of before. Ideally, you find 25 of them and arrange them in a nice 5×5 table. You know what you are doing :)

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[1] R is a constant that makes τ dimensionless, but nothing will depend on it