Introduction to string theory
Homework 3

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Problem 1 (Mass in light-cone coordinates)**.** Recall the solution of the classical equation of motion in the form

$$\begin{aligned} X^i(\tau,\sigma) &= X_L^i(\sigma^+) + X_R^i(\sigma^-)\ ,\ i = 1,\dots,D-2 \\ X_L^i &= \frac{1}{2}x^i + \frac{1}{2}\alpha' p^i \sigma^+ + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^i e^{-in\sigma^+} \\ X_R^i &= \frac{1}{2}x^i + \frac{1}{2}\alpha' p^i \sigma^- + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^i e^{-in\sigma^-} \\ X^+(\tau,\sigma) &= x^+ + \alpha' p^+ \tau\ . \end{aligned}$$

Consider a similar expansion as for X^i also for X^- and show that, up to one integration constant, it is completely given by the equation of motion $\partial_+\partial_- X^-$. Find the expression for the coefficients $\alpha_n^-, \tilde{\alpha}_n^-$ in terms of $\alpha_n^i, \tilde{\alpha}_n^i$ and p^+ for $n \neq 0$.

Consider what the expression for M^2 looks like in light coordinates and from the condition for p^- find the relation for the mass of the excited string in terms of $\alpha_n^i, \tilde{\alpha}_n^i$.