Introduction to string theory
Homework 6

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Problem 1 (Energy-momentum tensor in ghost CFT)**.** The action for the ghost fields turned out to be

$$S = \frac{1}{2\pi}\int d^2z\ \left(b\bar{\partial}c + \bar{b}\partial\bar{c}\right)\ .$$

Show that the energy momentum tensor is given by

$$T = 2(\partial c)b + c\partial b\ ,\ \bar{T} = 2(\bar{\partial}\bar{c})\bar{b} + \bar{c}\bar{\partial}\bar{b}\ .$$

Hint. Recall that when varying the (general) action with respect to metric there are all sorts of strange contributions from nonzero Christoffel symbols and tracelessness of $b_{\alpha\beta}$, which need to be taken care of.

Problem 2 (Field correlators in ghost CFT)**.** Using appropriate Ward identities show that the contractions / OPEs for the ghost fields are given by

$$b(z)c(w) = -c(w)b(z) = \frac{1}{z-w}\ ,\ bb = cc = 0\ .$$

Show, how the cb OPE follows from the first one and Fermi statistics of the ghost fields.

Hint. This is done in various degrees of explicicity in most of the string theory textbooks.

Problem 3 (Energy-momentum tensor OPE in ghost CFT)**.** Consider a slightly generalized energy-momentum tensor

$$T =: (\partial b)c : -\lambda\partial : bc :\ .$$

Show that b is a primary field with weight $h = \lambda$ and c is a primary field with $h = 1-\lambda$. Finally show that the central charge if such system is

$$-12\lambda^2 + 12\lambda - 2\ .$$

Hint. This is done in various degrees of explicicity in most of the string theory textbooks.