QUANTUM SPACETIMES & GRAVITY IN IKKT MM.

based on 2411.02598, 2502.02498 + ...
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Outline:

1. QUANTUM SPACETIMES
2. IKKT
3. 1-LOOP & GRAVITY

Take home message

IKKT as a quantum theory
of QUANTUM spacetimes

which describes (some) gauge & gravitional dynamics

1. Quantum Spacetimes

1.1. Why Quantum Spacetimes?

Quantum Gravity:

- QFT & QM we understand* small scale physics, particles, atoms, ...
- Einstein GRAVITY: large scale physics, planets, cosmology*, BHs*, galaxies*

All *'s might need QG.

- Problems w/quantizing gravity:
  - GR not renormalizable as a QFT:
    - UV completion?
    - GRAVITY as EMERGENT PROPERTY (e.g IKKT)
  - (RELATED) CAN'T LOCALIZE TOO MUCH (OR BH)

hep-th/0301037

$\rightsquigarrow$ INTRINSIC UNCERTAINTY IN COORDINATES GENERALIZING HEISENBERG UNCERTAINTY

$$\Delta X_\mu \, \Delta X_\nu \gtrsim \ell_p^2$$

$$\Downarrow$$

appears naturally across various QG approaches, STRING, LQG

$$[X_\mu, X_\nu] \neq 0$$

QUANTUM, NC, SPACETIME

1.2. NC SPACES

NC geometry is a rich math field, very complicated: we're interested in a simpler kind of spaces, motivated by the physical model (IKKT).

Rough general idea:

CLASSICAL SPACES $\xleftrightarrow{\text{1-to-1}}$ Algebra of functions on them $C(\mathcal{M})$

$\mathcal{M}$

$\downarrow Q$

NC SPACE? $:=$ NC Algebra of QUANTIZED FUNCTIONS, $\mathrm{End}(\mathcal{H})$

commutative limit.

· Quantization of symplectic mf's

$$Q : C(\mathcal{M}) \longrightarrow \mathrm{End}(\mathcal{H})$$

mapping P.B. $\to$ comm

+ nice properties

& semi-classical limit

(regime $\lambda \gg \theta$)

SIMPLEST: MW PLANE $\mathbb{R}^{2n}_\theta$

think of as pos. + momentum phase space quantization

QM!

$$\begin{array}{cc} q & p \end{array}$$

$$\{X^a, X^b\} = \begin{pmatrix} 0_n & id_n \\ -id_n & 0_n \end{pmatrix} \rightsquigarrow [X^a, X^b] = i\underbrace{\theta}_{\hbar} \begin{pmatrix} 0 & id \\ -id & 0 \end{pmatrix}$$

$X^a = q_1, \dots, q_n, p_1, \dots, p_n$

We usually look at quantized coadjoint orbits:

spaces constructed w/ some symmetry group action G admit quantizations $\mathrm{End}(\mathcal{H})$, $\mathcal{H}$ some unitary irrep of G.

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e.g. FUZZY SPHERES:

sphere $x^a x^a = 1$ $\quad a = 1,2,3$ $\quad (x^a : S^2 \hookrightarrow \mathbb{R}^3)$

can be constructed as $SO(3)$ coadjoint orbit

sympl structure $\omega_N = \frac{N}{4}\varepsilon_{abc}\, x^a dx^b \wedge dx^c$, $\{x^a, x^b\} = \frac{2}{N}\varepsilon^{abc} x^c$

$x^a \longrightarrow$ SYMM TRACELESS POLY'S in x^a

$\leadsto$ algebra of functions on S^2, generated by spherical harmonics

$$Y^l_m(x) = \underbrace{Y^{(m)}_{a_1 \dots a_l}}_{\text{sym traceless}} x^{a_1} \dots x^{a_l}$$

$|m| \leq l$ $\longrightarrow$ spin l $SO(3)$ irrep

$$\boxed{\mathcal{C}(S^2) = \bigoplus_{l=0}^{\infty} (l)}$$

FUZZY SPHERE S^2_N

$N\times N$ hermitian matrices $X^a \in \mathrm{End}(\mathcal{H})$, $\mathcal{H} = \mathbb{C}^N$ $\quad a=1,2,3$

J^a ang momentum gen. in spin $\left(\frac{N-1}{2}\right)$ rep

then $X^a := \dfrac{J^a}{\sqrt{C_N^2}}$, $C_N^2 = \frac{1}{4}(N^2-1)$

$$\hookrightarrow \left[X^a, X^b\right] = \frac{i}{\sqrt{C_N^2}} \varepsilon^{abc} X^c \quad , \quad \sum X^a X^a = id_N$$

CLAIM:

Quantized algebra of function on (S^2, ω_N), $\mathrm{End}(\mathcal{H})$ w/ $\langle \phi, \psi \rangle = \mathrm{Tr}(\phi^\dagger \psi)$, analog $SO(3)$ w/ $J^a \cdot \phi = [J^a_{(N)}, \phi]$

X^a QUANTIZED EMBEDDING COORDINATES

$$\mathrm{End}(\mathcal{H}) \simeq \left(\frac{N-1}{2}\right) \otimes \left(\frac{N-1}{2}\right)^* = \bigoplus_{\ell=0}^{N-1} (\ell)$$

spin ℓ irrep of $SO(3)$

$\rightsquigarrow$ "CUT-OFF" algebra of function on S^2

DEF FUZZY SPHERICAL HARMONICS AS TRACELESS, SYM, ...

$Q: \; Y^\ell_m \longmapsto \hat{Y}^\ell_m$, $\underline{\ell \text{ cutoff}}$

$$\sqrt{C^2_N} = \frac{\sqrt{N^2-1}}{2} \sim \frac{1}{\hbar}$$

i.e. $L^2_{NC} \sim \frac{1}{\sqrt{C^2_{NC}}} \sim \frac{2}{N}$ ~ good semiclassical approx when N large.

$$L_{UV} := N$$

cutoff.

1.4. Covariant Cosmological quantum spacetime

QUANTIZED COADJOINT ORBITS of $SO(4,1)$

constructed from $SO(4,2)$ gen $\mathcal{M}^{ab}$ $a,b = 0,1,\dots,5$

T^{1} $(\sim M^{1,4})$

quantization of a

$H^4 \tilde{\times} S^2$ bundle

4d spacetime

internal sphere

HIGHER SPIN MODES

1.3. PROBING THE GEOMETRY

Interpretation of X^a as quantized embedding functions.

$$\underset{\hat{\in}\,\mathrm{End}(\mathcal{H})}{X^a} = Q(x^a) \quad , \quad x^a : \mathcal{M} \hookrightarrow \mathbb{R}^D$$

· TOOL: define states in $\mathcal{H}$ that make sense of this

QUASI-COHERENT STATES

GIVEN MATRIX CONFIG X^a

for each $x^a \in \mathbb{R}^D$, define

$$H_x := \frac{1}{2} \sum (X^a - x^a)^2$$

distance from matrix config to point

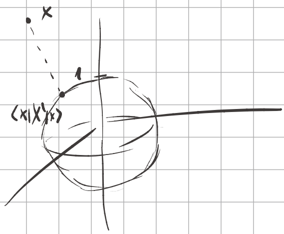
↳ minimize ⤳ ground state

$$H_x |x\rangle = \lambda(x) |x\rangle$$

then

$$\langle x| H_x |x\rangle = \underbrace{\sum (\Delta X^a)^2_{|x\rangle}}_{\text{uncertainty}} + \sum \underbrace{(\langle x|X^a|x\rangle - x^a)^2}_{\text{distance from point } x^a \text{ to exp value}}$$

idea, e.g. FUZZY SPHERE



$|x\rangle$ are states that localize X^a as much as possible on the geometry def. by them

More abstractly quantum space

$$\mathcal{M} = \{|x\rangle\} \big/ U(1) \hookrightarrow \mathbb{C}P^{N-1}$$

$$[|x\rangle] \longmapsto \langle x|X^a|x\rangle$$

embedded quantum space

geometric structures $\to$ quantum metric
sets scale on which coherent states are localized

– pre-symplectic form $\omega \to$ sets NC scale

Quantization map (coherent state quantization, analogous Weyl quant)

$$Q : \mathcal{C}(M) \to \mathrm{End}(\mathcal{H})$$

$$\phi(x) \mapsto \int_M \phi(x) |x\rangle\langle x|$$

$$\mathrm{Tr} \sim \int$$

$$[X^a, Q(\phi)] = Q(i\theta^{ab}\partial_b \phi)$$

$$\Phi \mapsto \langle x|\Phi|x\rangle = \phi(x)$$

"approx isometry" wrt HS norm on $\mathrm{Loc}(\mathcal{H}) \subset \mathrm{End}(\mathcal{H})$
and $\mathcal{C}_{IR}(M)$

almost local

$$\mathrm{Loc}(\mathcal{H}) \leftarrow \mathcal{C}_{IR}(M)$$

$$[\phi, \psi] \approx i\{\phi, \psi\}$$

on which ϕ, ψ approx commute

IR functions, probed by wave packets

$$\lambda \gg L_{NC}$$

1.5 NC FIELD THEORY

INSTEAD of Theory of fields in $\mathcal{C}(M)$
$\rightsquigarrow$ on $\mathrm{End}(\mathcal{H})$

On $\mathrm{Loc}(\mathcal{H}) \sim \mathcal{C}_{IR}$ similar to field theory
-amount modes, deep quantum regime
non-local!
that's what creates problems in NCFT

ACTIONS

$$S = \mathrm{Tr}\Big(-\tfrac{1}{2}\underbrace{[X^a,\phi][X_a,\phi]}_{\gamma^{ab}\partial_a\phi\,\partial_b\phi} + \tfrac{1}{2}m^2\phi^2\Big)$$

$$\gamma^{ab} = \theta^{ac}\eta_{cd}\theta^{db} \rightarrow \boxed{\text{metric}}!!$$

$\mathrm{tr} \rightarrow \int \rightarrow$ field theory

Remark "Physical metric"

$\mathrm{tr} \rightarrow \int \rho_M \Rightarrow$ $\boxed{\rho_M \gamma^{ab} = \sqrt{|G|}\, g^{ab}}$

symplectic density

PROBLEM:

1-loop ⇝ QFT diagram

1-loop ⇝ NON-PLANAR

non-local contribution

NO COMM. COUNTERPART

UV-IR MIXING

non-local modes running in loops!

NCFT ⇝ NOT JUST A CUTOFF of QFT.

SYMPLECTOMORPHISMS

$$\phi \to U^{-1}\phi U \quad , U = e^{i\Lambda} \quad \delta_\Lambda \phi = -i[\Lambda, \phi]$$

$$\delta\phi = \{\Lambda, \phi\} = \xi^\mu \partial_\mu \phi$$

$$\{\Lambda, \ \} = \xi^\mu \partial_\mu$$

GAUGE YM

$$S = \frac{1}{g^2} \mathrm{Tr}\left([T^a, T^b][T_a, T_b]\right)$$

$T^a \in \mathrm{End}(\mathcal{H}) \otimes M(K)$,

on Q SPACE X^a,

$$T^a = X^a + A^a$$

easy to see, $A^a = \theta^{ac} A_c$, A_c ∈ $\mathrm{End}(\mathcal{H}) \otimes M_n$

gauge theory from fluctuations of Q SPACE

$$S \simeq \frac{-1}{g^2} \int \frac{\Omega}{(2\pi)^n} \mathrm{Tr}\left(F_{ab} F^{ab}\right)$$

$F_{ab} F^{ab}$: contracted

- can add fermions
- $M(K)$ gauge inv.

$$\left(g^2_{YM} = g^2 L_{NC}^{2n-8} \right)$$

$U(1)$ sector $\Lambda = \Lambda_0 \mathrm{id}$ non-trivial $\rightsquigarrow$ symplectic diffeos

1.8 GEOMETRY

(SKETCH)

we now that we can extract metric

meaning of T^a & $[T^a, T^b] \overset{= -i f^{ab}}{} ?$

$\{T^a, x^\mu\} \sim E^{a\mu}$ (rescaled frame)

$\left(\mathrm{tr}[T_a, \phi][T^a, \phi] \sim \int \Omega\, \partial_\mu \phi\, \partial^\mu \phi \right.$

$T^{ab\mu} := \{\theta^{ab}, x^\mu\} \to$ $\overset{\text{TORSION}}{\text{Weitzenbock}}$ connection

$(R^{\mu}{}_{\nu\rho\sigma} = 0$, parallel to $E)$

Saw earlier metric $G \sim \rho\,\theta\,\eta\,\theta$

COV. QUANTUM SPACETIME

$\downarrow$

$K = -1$ FLRW metric

2. IKKT

$$Z = \int dX d\psi \, e^{iS}$$

$$S = \frac{1}{g^2} \mathrm{Tr}\left([X^a, X^b][X_a, X_b] + \bar{\psi} \Gamma_a [T^a, \psi] \right)$$

$a = 0, \dots, 9$ $T^a \in \mathrm{End}(H)$ ψ MW spinors of $SO(9,1)$

2.1 History

NG action string worldsheet $\longrightarrow$ SCHILD ACTION $\boxed{IIB}$

CAN BE GAUGE FIXED TO BE

$$S_{Schild} \sim \int d^2\sigma \left[\{X^a, X^b\}^2_P + \bar{\psi} \Gamma_a \{X^a, \psi\}_P \right]$$

$$\{\phi, \psi\} = \frac{1}{2} \epsilon^{\alpha\beta} \partial_\alpha X^a \partial_\beta X^b$$

"MATRIX REGULARIZATION" $\{ \ , \ \} \longmapsto i[\ ]$

Vertices $V \sim \mathrm{tr}\, e^{ik_a X^a}$ + SUSY PARTNERS

$\updownarrow$

MASSLESS MULTIPLET OF TYPE IIB SUGRA

- just matrix integral

PROPOSED AS NP def of IIB

2.2 The model

- $SO(1,9)$ inv.
- $U(N)$
- SUSY ~> dim reduction of $\mathcal{N}=1$ 10d SYM or $\mathcal{N}=4$ SYM 4d | hep-th/9612115

- As explained :

X^a as NC embedding brane $\hookrightarrow \mathbb{R}^{1,9}$

- We focus on weak coupling on NC branes
no holography, physics on brane.

EOM $\Box X^a = \bar{\Psi}\Gamma^a\Psi$, $\Box = -[X^a,[X_a,\cdot]]$
$\cancel{D}\Psi = 0$ $\qquad \cancel{D} = \Gamma^a[X_a,\cdot]$

- NON-TRIVIAL solutions are
fuzzy spaces

Quantum theory (quantum partition function)
of QUANTUM SPACES

Fun facts

Intriguing numerical results: Evidence for 1-loop stability for backgrounds w/ 4 large & 6 small dimensions i.e. appears to predict the dimensionality of spacetime!

SUSY $\rightsquigarrow$ from MAX $\mathcal{N}=4$ SUSY

$\rightarrow$ tames UV/IR MIXING, divergences cancel

$\rightarrow$ this model is "unique" among NCFT in niceness.

Naturally contains: 10d $\mathcal{N}=1$ SYM $\quad T^a = X^a + A^a$

. 4d $\mathcal{N}=4$ SYM $\quad T^a = \begin{cases} X^\mu + A^\mu, & \mu=0,\dots,3 \\ \phi^i & i=4,\dots,9 \end{cases}$

X^μ NW flat

$\downarrow$

NC SYM !

3. GRAVITY at ONE-LOOP

3.1 GRAVITY

Look at generic backgrounds $T^a = \begin{cases} T^i & \sim M_4 \\ K^i & \sim K_6 \end{cases}$

$$Z = e^{i\Gamma_{eff}}$$

$$\hookrightarrow \Gamma_{1\text{-loop}} = -\frac{i}{2}\int_0^\infty \frac{d\lambda}{\lambda}\, \mathrm{tr}\, e^{-i\lambda\Box}\, Q_{10}$$

$= \mathrm{tr}\log([\Box - [\cdot,\cdot]]) + \ldots$

$\mathrm{tr}_{\mathrm{End}(\mathcal{H})\otimes\mathbb{R}}$

sum over field fluctuations

$A \in \mathrm{End}(\mathcal{H}_r) \otimes \mathrm{End}(\mathcal{H}_K)$

Q_{10}: $SO(1,9)$ character

comb. of $e^{i\lambda\Sigma^{(r)}_{ab}[[T^a,T^b],\cdot]}$

· A lot to unpack:

· decompose 10_d Rep (and tr) into $SO(1,3)$ & $SO(6)$

· expand 4d tr $e^{i\lambda\Sigma_{ab}[[T^a,T^b],\cdot]}$ (we skip justification)

$$Q_{10} \approx X_6 + \mathrm{tr}\lambda^2 [[T^a,T^b],\cdot]^2\, G_6 + O(\lambda^4)$$

X_6, G_6 $SO(6)$ characters

$X_6 \rightsquigarrow$ potential for extra dim KK modes

we saw $\mathrm{tr}_{\mathrm{End}\,\mathcal{H}} [[T^\mu, T^\nu], \cdot\,] \sim \boxed{\text{torsion}}$

$\mathrm{End}(\mathcal{H})$

$\mathrm{tr} \leftrightarrow \int$

capture non-local modes $\longleftarrow$ STRING MODES $|x\rangle\langle y|$

↳ SHORT $\to$ GAUSSIAN WAVE PACKETS $(|x-y| < L_{NC}$

$\lambda \sim L_{NC})$

$\psi_{\lambda x}$ – LONG $(|x-y| > L_{NC})$

$\int d^4x \int d^4k \; T^{\mu\nu\rho} k_\rho \psi_{k,x}$

$$S_{1\text{-loop}} \supset \int \underbrace{T^{\mu\nu\rho} T_{\mu\nu\rho}}_{} \; \frac{G_6}{\rho^2} \; dvol$$

ρ: scalar coming from sympl density

$= R + (\dots)$

$(\dots)$: torsion, dilaton, ...

$\supset S_{EH}$ w/ $G_N \sim \dfrac{\rho^2}{G_6}$

$G_6 \to$ internal geom FUZZY

α integral $G_6[\alpha]$

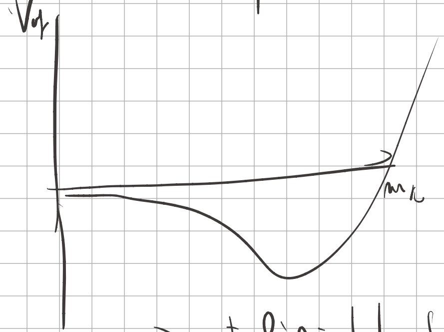
• NEED EXTRA DIMENSIONS for 4D GRAVITY!

3.2. POTENTIAL

for T^7 = COSMO QUANTUM SPACETIME

$$\mathrm{tr}\, X_6 \rightsquigarrow V(\underbrace{m_\kappa}_{KK})$$

For suitable parameter ranges



→ stabilized! Start from $\overset{nc}{\mathcal{N}=4}$ SYM

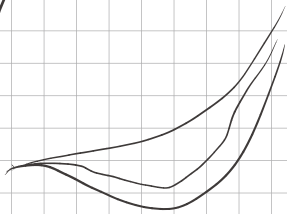
(superconformal, no scale, very rigid)

induced scale! (PHYSICAL CON)

CAVEATS :

↳ T^m → matrix/IKKT (NO SUSY)

↳ at late times



NO MINIMUM

PHYSICAL COW BECOMES SPHERICAL

BOTH PROBLEMS SEEM TO BE FIXED for DYNAMICAL deformations

$$\begin{cases} \tilde{T}^m = \alpha(\tau) T^m \\ \tilde{K}^A = f(\tau) K^A \end{cases}$$

COSMO. Q. SPACETIME SOLUTIONS

$$\Box T^{\mu} = \frac{3}{R^2} T^{\mu}$$ mass deformation of IKKT

SO(3,1) covariant

?

$$\tilde{T}^{\mu} = \alpha(\tau) T^{\mu}$$ ⟶ can solve IKKT eom

└ COSM CONF TIME

↓ one step further

$$\begin{cases} \tilde{T}^{\mu} = \alpha(\tau) T^{\mu} \\ \tilde{K}^{I} = \underbrace{f(\tau)}_{} K^{I} \end{cases}$$

dynamical EXTRA DIM.

- STABILIZATION of (SPINNING) EXTRA DIM

- RICHER FAMILY of $K=-1$ FLRW GEOMETRIES for $\mathbb{R}^{3,1}$ → SENSIBLE 1-loop?

Rmk

From same repr. can construct $K=0$ FLRW geometries

SUMMARY & OUTLOOK

- QUANTUM SPACETIMES as playground for QUANTUM GRAVITY
  $\rightsquigarrow$ COV. COSM. Q. SPACETIME RICH STRUCTURE,

- IKKT AS QUANTUM THEORY for QUANTUM SPACETIMES
  - GAUGE (tree level)
  - GRAVITY $\supset S_{EH}$ 1-loop ← need extra dims!!

seem like they can dynamically stabilize!!

OUTLOOK:
- DYNAMICAL deformations at 1-loop
- $K=0$
- WORK OUT all the details of 1-loop physics

  ↓

  GRAVITY & GAUGE

  ↓

  REALISTIC? IF SO, COSMO IMPRINTS?
- more general properties of IKKT?