① $S(E,V,N) = 3a E^{1/3} V^{1/2} N^{1/6}$

a) $\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V,N} = 3a\frac{1}{3} E^{-2/3} V^{1/2} N^{1/6} \Rightarrow T(E,V,N) = \frac{E^{2/3}}{a V^{1/2} N^{1/6}}$

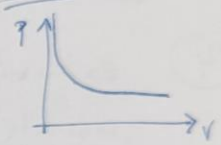
$\frac{p}{T} = \left(\frac{\partial S}{\partial V}\right)_{E,N} = 3a E^{1/3} \frac{1}{2} V^{-1/2} N^{1/6} \Rightarrow p(E,V,N) = \frac{E^{2/3}}{a V^{1/2} N^{1/6}} \cdot 3a E^{1/3} \frac{1}{2} V^{-1/2} N^{1/6} = \frac{3}{2}\frac{E}{V}$

$-\frac{\mu}{T} = -\left(\frac{\partial S}{\partial N}\right)_{E,V} = -3a E^{1/3} V^{1/2} \frac{1}{6} N^{-5/6} \Rightarrow \mu(E,V,N) = -\frac{E^{2/3}}{a V^{1/2} N^{1/6}} 3E^{1/3} V^{1/2} \frac{1}{6} N^{-5/6} = -\frac{1}{2}\frac{E}{N}$

b) $T(E,V,N) = \frac{E^{2/3}}{a V^{1/2} N^{1/6}} \Rightarrow E = (aTV^{1/2}N^{1/6})^{3/2} = (aT)^{3/2} V^{3/4} N^{1/4}$

$p(E,V,N) = \frac{2}{3}\frac{E}{V} \Rightarrow \frac{3}{2}\frac{1}{V}(aT)^{3/2} V^{3/4} N^{1/4} = \frac{3}{2}(aT)^{3/2}\left(\frac{N}{V}\right)^{1/4} = p(T,V,N)$

izoterma: $p(T=\text{const}, V, N=\text{const.}) \sim V^{-1/4}$ $\quad pV^{1/4} = \text{const}$



c) $A = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} p(T,V,N)\,dV = \frac{3}{2}(aT)^{3/2} N^{1/4} \int_{V_1}^{V_2} V^{-1/4}\,dV = 2(aT)^{3/2} N^{1/4}\left(V_2^{3/4} - V_1^{3/4}\right)$

d) $dE = TdS - pdV$ → izochora … $dV = 0$

$dE = TdS$

$dS = \frac{dE}{T}$ $\quad E(T,V,N) = (aT)^{3/2} V^{3/4} N^{1/4} \Rightarrow dE = a^{3/2} V^{3/4} N^{1/4} \frac{3}{2} T^{1/2}\,dT$

$\Delta S = \int_{T_1}^{T_2} dS = \int_{T_1}^{T_2} \frac{dE}{T} = \int_{T_1}^{T_2} a^{3/2} V^{3/4} N^{1/4} \frac{3}{2} T^{-1/2}\,dT = 3a^{3/2} V^{3/4} N^{1/4}\left(T_2^{1/2} - T_1^{1/2}\right)$

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② $S(E,V,N) = 3a E^{1/3} V^{1/2} N^{1/6}$

a) $E = \left(\frac{S}{3aV^{1/2}N^{1/6}}\right)^3 = E(S,V,N)$

b) $F = E - TS = E - \frac{E^{2/3}}{a V^{1/2} N^{1/6}} 3a E^{1/3} V^{1/2} N^{1/6} = E - 3E = -2E =$

$= -2(aT)^{3/2} V^{3/4} N^{1/4}) = F(T,V,N)$

c) $G = E - TS + pV = F + pV = -2E + \frac{3}{2} = -\frac{1}{2}E$

$p(T,V,N) = \frac{3}{2}(aT)^{3/2}\left(\frac{N}{V}\right)^{1/4} \Rightarrow V^{1/4} = \frac{3}{2}(aT)^{3/2} N^{1/4}\frac{1}{p}$

$V = \left(\frac{3}{2}\right)^4 (aT)^6 N p^{-4}$

$E(T,V,N) = (aT)^{3/2} V^{3/4} N^{1/4} \Rightarrow E(T,p,N) = (aT)^{3/2}\left(\frac{3}{2}\right)^3 (aT)^{3/2} N^{3/4} N^{1/4} p^{-3} =$

$= a^6 T^6 \frac{27}{8} N p^{-3}$

$G = -\frac{1}{2}E = -a^6 T^6 \frac{27}{16} N p^{-3} = G(T,p,N)$

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(3) $\left(\frac{\partial u}{\partial x}\right)_{T,N} = -\left(\frac{\partial p}{\partial N}\right)_{x,T}$ ... $p = \frac{\partial(\pm \text{dajaký potenciál})}{\partial V}$

máme premenné V, T, N

... je to F! $(F(T,V,N))$

naotaj

$\left(\frac{\partial^2 E}{\partial V \partial N}\right)_T = -\left(\frac{\partial p}{\partial N}\right)_{V,T} = \left(\frac{\partial u}{\partial V}\right)_{T,N}$ ... $x = V$

---

(4) extenzivita $S(\lambda E, \lambda V, \lambda N) = 3a \lambda^{1/3} E^{1/3} \lambda^{1/2} V^{1/2} \lambda^{1/6} N^{1/6} = \lambda^1 S(E,V,N)$ ✓

stabilita $\frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1$

- $\left(\frac{\partial^2 S}{\partial E^2}\right)_{V,N} = 3a V^{1/2} N^{1/6} \frac{\partial^2 E^{1/3}}{\partial E^2} \sim \frac{1}{3}\frac{\partial E^{-2/3}}{\partial E} = -\frac{2}{9} E^{-5/3} < 0$ ✓
- $\left(\frac{\partial^2 S}{\partial V^2}\right)_{E,N} = 3a E^{1/3} N^{1/6} \frac{\partial^2 V^{1/2}}{\partial V^2} \sim \frac{1}{2}\frac{\partial V^{-1/2}}{\partial V} = -\frac{1}{4} V^{-3/2} < 0$ ✓
- treba veta TD: $\lim_{T\to 0} S = \lim_{T\to 0} E^{1/3}(\ldots) = \lim_{T\to 0} T^{1/2}(\ldots) = 0$ ✓
- rast S vs E $\left(\frac{\partial S}{\partial E}\right) = \frac{1}{T} = a E^{-2/3} V^{1/2} N^{1/6} > 0$ ✓

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(5)

$\mu_1(T,p) = \mu_{10} + v_1 p - \alpha T^2$

$\mu_2(T,p) = \mu_{20} + v_2 p - k_B \frac{T^4}{\Theta_D^3}$ $\quad v_1 > v_2$

... koexistujú pri $p_0, T = 0$ ... nájdeme p_0

$\mu_1(T,p) = \mu_2(T,p)$

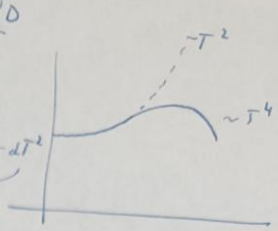
$\mu_{10} + v_1 p - \alpha T^2 = \mu_{20} + v_2 p - k_B \frac{T^4}{\Theta_D}$

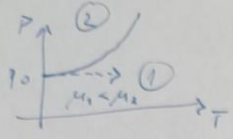
pri $T = 0$ $\quad p = \frac{\mu_{20} - \mu_{10}}{v_1 - v_2} = p_0$

- krivka koexistencie $p = p_0 + \frac{\alpha T^2 - 4\sigma T^4/\theta_D^3}{v_1 - v_2}$

- pre malé T prevládne člen $\sim T^2$

- pre malé T $T^2 \gg T^4 \Rightarrow$ $\mu_1(p_0, T \to 0) = \mu_2(p_0, 0) - \alpha T^2$





tj. nárast teploty z bodu $(p_0, T=0)$ zvyšuje μ_1 oproti μ_2 ... stabilizuje fázu jedna ① v oblasti pod grafom

- teplo $\quad s_1 = -\left(\frac{\partial \mu_1}{\partial T}\right)_p = 2\alpha T \qquad s_2 = -\left(\frac{\partial \mu_2}{\partial T}\right) = 4\sigma_B \left(\frac{T}{\theta_D}\right)^3$

$$q = T(s_1 - s_2) = 2\alpha T^2 - 4\sigma_B \frac{T^4}{\theta_D^3}$$

$q(T \to 0) \approx 2\alpha T^2 > 0$ ... teplo sa uvoľní do rezerv.

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6. $\quad e = pv \qquad p^2 v = (aT)^3$

$$p^2 v = (aT)^3$$

$$\frac{1}{p^2 v} = \frac{1}{a^3 T^3}$$

$$\frac{1}{T} = \left(\frac{a^3}{p^2 v}\right)^{1/3} = \left(\frac{a^3 v}{p^2 v^2}\right)^{1/3} = \left(\frac{a^3 v}{e^2}\right)^{1/3} = a\, v^{1/3} e^{-2/3}$$

$$\frac{p}{T} = p \cdot a v^{1/3} e^{-2/3} = e \cdot v^{-1} a\, v^{1/3} e^{-2/3} = a\, e^{1/3} v^{-2/3}$$

$$ds = a v^{1/3} e^{-2/3} de + a v^{-2/3} e^{1/3} dv$$

$$ds = d(3a v^{1/3} e^{1/3}) \Rightarrow \qquad s(e, v) = 3a v^{1/3} e^{1/3} + s_0$$

3. Veta $\Rightarrow s_0 = 0$

a) $S(EVN) = N\, s\left(\frac{E}{N}, \frac{V}{N}\right) = 3aN \frac{V^{1/3}}{N^{1/3}} \frac{E^{1/3}}{N^{1/3}} = 3a(NVE)^{1/3}$

b) $G = N\mu$

$$\frac{\mu}{T} = -\left(\frac{\partial s}{\partial N}\right)_{E,V} = -aN^{-2/3}(VE)^{1/3} \Rightarrow \mu = -aN^{-2/3}\left(\frac{N}{V}\right)^{1/3}\left(\frac{E}{N}\right)^{2/3} V^{1/3} E^{1/3} = -\frac{E}{N}$$

$$G = -E = -pvN = -p \cdot \frac{(aT)^3}{p^2} N = -\frac{(aT)^3}{T} N = G(T, p, N)$$

$p^2 v = (aT)^3$

$v = \frac{(aT)^3}{p^2}$

c) $\mu = -\frac{E}{N}$

$T = a^{-1} V^{-1/3} e^{2/3} = \frac{E^{2/3}}{a V^{1/3} N^{1/3}}$

$E = TS - pV + \mu N = \frac{E^{2/3}}{a V^{1/3} N^{1/3}} \cdot 3a (EVN)^{1/3} - 2E = 3E - 2E = E$

($pV \to E$, $\mu N \to E$)