Quantum Structure of Spacetime and Matrices

Planck scale

A clue for the scale of fundamental theory that is capable of encompassing both general relativity and quantum mechanics is given by the combinations of fundamental constants called the Planck units:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35}\,\text{m},$$
$$E_P = \sqrt{\frac{\hbar c^5}{G}} \approx 1.2 \times 10^{28}\,\text{eV}.$$

These are well above the reach of current technologies; for example, the energies probed at CERN's LHC are of the order of $10^{-16} E_P$.

Fuzzy physics

The structure of spacetime is expected to be discrete, or quantum, with a minimal length scale at the order of l_P.

A possible construction of such a space uses the idea from quantum mechanics — nontrivial commutators lead to uncertainty constraints:

$$[\hat{x}, \hat{y}] \neq 0 \;\Rightarrow\; \Delta x \Delta y > 0.$$

This is usually referred to as noncommutative geometry. The space retains continuous symmetries despite having a minimal length.

We study the properties of various physical systems, such as scalar fields and bounded quantum-mechanical systems, defined in such spaces and look for the effects of noncommutative construction on their properties.

This can be viewed as a toy-model for testing the consequences of quantum structure of spacetime on real physics.

References

[1] Benedek Bukor and Juraj Tekel.
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[2] Samuel Kováčik.
Hawking-radiation recoil of microscopic black holes.
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The fuzzy onion

A quantum space can be constructed using matrices (so-called fuzzy spaces); there is a correspondence between matrices and fields with limited spatial resolution. A well-studied example is the fuzzy sphere:

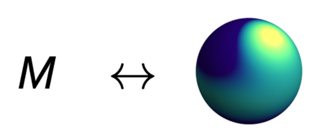

Figure 1. Hermitian matrices encode field configurations on a sphere; the size of spatial details is inversely proportional to the matrix size.

We have proposed a model for connecting the fuzzy spheres into a three-dimensional space [3]. This allows us to study real-world physics in quantum space or the properties of granular systems or systems with limited spatial resolution.

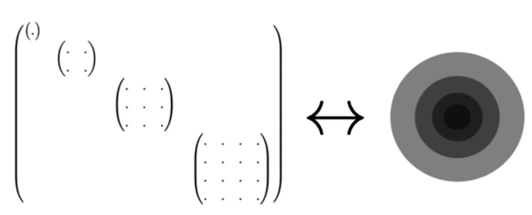

Figure 2. A model of three-dimensional quantum space using a particular form of matrix that describes a set of spheres of increasing radius with an onion-like structure.

Tools

Random matrices:
Analytical calculations are done using tools of random matrix theory. The relevant probability distributions are governed by the action of the theory, and the problematic part comes from the kinetic term. The challenging aspect is to analyze the asymmetric eigenvalue distributions properly.

Numerical simulations: The models we study are often expressed in terms of Hermitian matrices. While we are usually interested in studying the large-matrix limit, the behaviour settles for matrix size $N < 100$. Therefore, the mean value of observables can be approximated using numerical simulations, for example, using the Hamiltonian Monte Carlo or Matrix Bootstrap method. We study how these algorithms can be improved for the matrix models used to describe fuzzy spaces.

Phenomenology

Small black holes:
According to the standard theory of gravity, small black holes evaporate quickly due to Hawking radiation. However, the quantum structure of space stops this process as the size of the black hole approaches the Planck scale.

The resulting black hole is small enough to be virtually undetectable but still massive enough to be relevant even with modest concentration across astronomical scales. Therefore, small black holes are a possible dark matter candidate. We have shown that this behaviour holds for a large class of models but leads to a Hawking-radiation recoil effect that warms the dark matter and leads to strong restrictions on the formation of small black holes — shortly after the cosmological inflation era [2].

Vacuum dispersion:
The quantum structure of space modifies the momentum dispersion law. For light, this means that different wavelengths travel with different velocities, even in a vacuum. This effect is inversely proportional to the Planck energy; therefore, its relative size is of the order of 10^{-20} even for gamma radiation. However, signals from gamma-ray bursts can have flight time around the order 10^{20} s, making this effect observable given sufficient precision is obtained:

$$E^2(p) = p^2c^2\left(1 - 3pc/E_P + \mathcal{O}\left(E_P^{-2}\right)\right).$$

We have shown that the model of three-dimensional quantum space we studied leads to results consistent with other approaches (such as Lorentz-invariance violation). This model also leads to threshold anomaly, which can explain the large number of >10 TeV photons detected from the GRB221009A burst.

Bounded states:
The quantum structure of space can ever so slightly change the properties of quantum mechanical systems, such as their energy levels. These changes then possibly translate into differences in measurable quantities. One such example is a bound state of two quarks observed at particle accelerators and fuzzy correction to its mass [1]. In this particular case, the difference is too small to be distinguished at the moment, but it demonstrates the possibility of identifying such effects.

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