Cvičenie 13

Prepočítané príklady

- 11.7
- 11.12, 11.13, tenzor deformácie pre pole posunutí $\vec{u}(\vec{r},t) = ky\vec{e}_1$
- 11.19

Domáca úloha

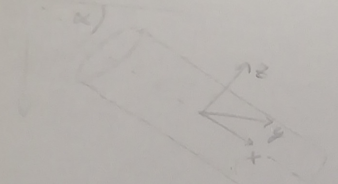
- Dokončenie výpočtu v príklade 11.9 pre rúru naklonenú o uhol α (na konci pdf)
- 11.8 (na konci pdf)
- 11.11
- 11.17, 11.18
- Dokončiť príklad na poruchovú metódu z cvičenia (na konci tohto pdf)
- Príklady zo stránky druhého cvičiaceho
- Zopakovať si variačný počet (príklady na konci tohto pdf)

Treba si zapamätať

- Pohybová rovnica kontinua: $f_i + \frac{\partial \sigma_{ij}}{\partial x_j} = \rho a_i(\vec{r},t)$
- Eulerova rovnica: $(\vec{v} \cdot \vec{\nabla})\vec{v} + \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g}$
- Bernoulliho rovnica: $\frac{1}{2}\rho v^2 + p + \rho g z = const.$
- Rovnica kontinuity: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$
- Navier-Stokesova rovnica: $(\vec{v} \cdot \vec{\nabla})\vec{v} + \frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \frac{\eta}{\rho}(\vec{\nabla}(\vec{\nabla} \cdot \vec{v}) + \triangle \vec{v})$
- Tenzor deformácie: $\epsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$

11.9 [illegible] v naklonenej rúre s kruhovým prierezom

α)



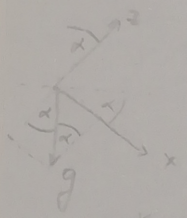
$$\vec{g} = -g\cos\alpha\,\vec{e}_z + g\sin\alpha\,\vec{e}_x$$

Navier-Stokesova rovnica

$$(\vec{v}\cdot\vec{\nabla})\vec{v} + \partial_t\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \frac{\eta}{\rho}\left(\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + \Delta\vec{v}\right)$$

Rovnica kontinuity

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$



1) nestlačiteľná $\Rightarrow$ ρ konšt. $\qquad \xRightarrow{RK}$ $\operatorname{div}\vec{v} = 0$

2) ustálené prúdenie $\Rightarrow$ $\partial_t\vec{v} = 0$

$\Rightarrow (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \frac{\eta}{\rho}\Delta\vec{v}$

ansatz: $\vec{v} = v(y,z)\,\vec{e}_x$

↳ platí R.K.? $\quad \operatorname{div}\vec{v} = \partial_x v(y,z) = 0$ ✓

↳ N.S.: $\quad v(y,z)\,\partial_x v(y,z) = 0 = (\vec{v}\cdot\vec{\nabla})\vec{v}$

$$\Delta\vec{v} = \left(\partial_y\partial_y v(y,z) + \partial_z\partial_z v(y,z)\right)\vec{e}_x$$

$\Rightarrow$

$$0 = -\frac{1}{\rho}\partial_x p + g\sin\alpha + \frac{\eta}{\rho}\Delta v \xrightarrow{3} -g\sin\alpha + \frac{1}{\rho}g'(x) = \frac{\eta}{\rho}\left(\partial_y\partial_y v(y,z) + \partial_z\partial_z v(y,z)\right)$$

$$0 = -\frac{1}{\rho}\partial_y p \xrightarrow{1} p = f(x,z)$$

$$0 = -\frac{1}{\rho}\partial_z p - g\cos\alpha \xrightarrow{2} p = -\rho g\cos\alpha\, z + g(x)$$

$\Rightarrow$

$$\underbrace{-g\sin\alpha + \frac{1}{\rho}g'(x)}_{\text{funkcia } x} = \underbrace{\frac{\eta}{\rho}\left(\partial_y\partial_y v(y,z) + \partial_z\partial_z v(y,z)\right)}_{\text{funkcia } y,z} \stackrel{!}{=} k$$

$\Rightarrow \quad g'(x) = \rho k + \rho g\sin\alpha \longrightarrow g(x) = \rho k x + \rho g\sin\alpha\, x + C$

$$\partial_y\partial_y v(y,z) + \partial_z\partial_z v(y,z) = \frac{\rho}{\eta}k = \Delta v$$

$\Rightarrow p(x,y,z) = -\rho g\cos\alpha\, z + (\rho k + \rho g\sin\alpha)\,x + C$

$\Rightarrow \Delta v(y,z) = \frac{\rho k}{\eta} \qquad$ okrajová podmienka $\left. v(y,z)\right|_{y^2+z^2=R^2} = 0$ ... stačí na stenách rúry (kvôli [illegible]

↳ polárne súradnice $\quad y = r\cos\varphi, \; z = r\sin\varphi \qquad \varphi \in (0, 2\pi>$

$r \in (0, R>$

↳ $(\partial_x^2 + \partial_y^2 + \partial_z^2) = \frac{1}{r}\partial_r(r\partial_r) + \frac{1}{r^2}\partial_\varphi^2 \qquad$ OP: $\left. v(r,\varphi)\right|_{r=R} = 0$

↳ symetria v φ (ani OP nezávisí od φ, čakáme to aj od riešenia)

$$\frac{1}{r}\partial_r(r\partial_r)v(r) = \frac{\rho k}{\eta} \qquad v(R) = 0$$

$$\partial_r(r\partial_r v) = \frac{\rho k}{\eta}r \longrightarrow r\partial_r v = \frac{\rho k}{\eta}\frac{r^2}{2} + C_1 \longrightarrow \partial_r v = \frac{\rho k}{\eta}\frac{r}{2} + \frac{C_1}{r}$$

$$\longrightarrow v(r) = \frac{\rho k}{\eta}\frac{r^2}{4} + C_1\ln r + C_2 \Rightarrow v(R) = 0 = \frac{\rho k R^2}{\eta\, 4} + C_1\ln R + C_2$$

↳ pretože nechceme nekonečné rýchlosti

$$v(0) = c_1 \underbrace{\ln 0}_{\infty} + c_2 = \text{konečné} \Rightarrow c_1 \stackrel{!}{=} 0 \Rightarrow c_2 = -\frac{\rho k R^2}{4\eta}$$

$$\hookrightarrow v(r,\varphi) = \frac{\rho k}{4\eta} r^2 - \frac{\rho k R^2}{4\eta} = \frac{\rho k}{4\eta}(r^2 - R^2) = \frac{\rho k}{4\eta}(y^2+z^2-R^2) = v(y,z)$$

$$\Rightarrow v(x,y,z) = \frac{\rho k}{4\eta}(y^2+z^2-R^2)$$

$$p(x,y,z) = -\rho g \cos\alpha\, z + \rho x(k + g\sin\alpha) + C$$ → C udáva tlak v počiatku

⇒ konštanty k, C

→ pre $\alpha = 0$ … $p = -\rho g z + \rho k x + C$

$$p(0,0,0) = p_A + \Delta p = C$$

$$p(L,0,0) = p_A = \rho k L + p_A + \Delta p \Rightarrow k = -\frac{\Delta p}{\rho L}$$ ⇒ k súvisí s pretlakom

$$\Rightarrow p = -\rho g z - \Delta p \frac{x}{L} + p_A + \Delta p = p_A + \Delta p\left(1 - \frac{x}{L}\right) - \rho g z$$

→ ak pre nulový tlak dostaneme:
↳ izobary

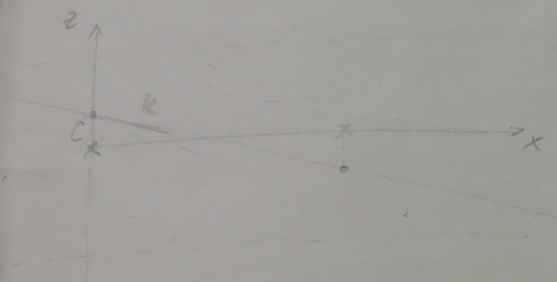
$$\rho g \cos\alpha\, z = \rho x(k + g\sin\alpha) + C$$

$$z = \frac{k + g\sin\alpha}{g\cos\alpha} x + \frac{C}{\rho g \cos\alpha}$$

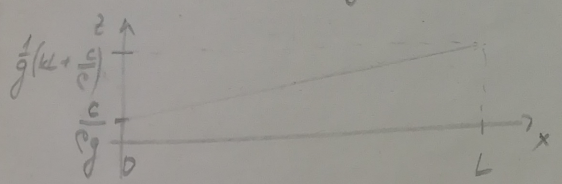
⇓

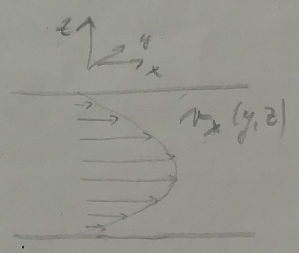
k udáva sklon hladiny s nulovým tlakom v rovine xz

C udáva z-pozíciu hladiny s nulovým tlakom v mieste $x=0$



$$\alpha = 0 \qquad z = k\frac{x}{g} + \frac{C}{\rho g}$$

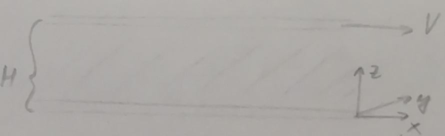




11.8 $\vec{v}(\vec{r})$ – ustálený stav pre pohyb viskóznej kvapaliny medzi stojacim dnom a vodorovnou doskou ťahanou rýchlosťou V v smere x

Navier-Stokesova rovnica:

$$(\vec{v}\cdot\vec{\nabla})\vec{v} + \partial_t \vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \frac{\eta}{\rho}\left(\vec{\nabla}(\vec{\nabla}\cdot\vec{v}) + \Delta\vec{v}\right)$$



- ustálený stav $\quad \partial_t \vec{v} = 0$
- $\vec{g} = -g\,\vec{e}_x$
- nestlačiteľná $\Rightarrow$ $\rho =$ konšt. $\Rightarrow$ rovnica kontinuity: $\mathrm{div}\,\vec{v} = 0$

$\Rightarrow$ hľadáme $\vec{v}(\vec{r})$, $p(\vec{r})$ = 4 funkcie

máme Navier-Stokesovu rovnicu (3), rovnicu kontinuity (1) = 4 rovnice

① $\mathrm{div}\,\vec{v} = 0$

② $(\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \frac{\eta}{\rho}\Delta\vec{v}$

Ansatz: $\vec{v} = v\,\vec{e}_x$, lebo dosku ťaháme v smere x

$\Rightarrow (\vec{v}\cdot\vec{\nabla})\vec{v} = (v\partial_x)\vec{v} = \vec{e}_x(v\partial_x)v$

$\Delta\vec{v} = \Delta v\,\vec{e}_x$

$\Rightarrow \vec{e}_x$: $\quad (v\partial_x)v = -\frac{1}{\rho}\partial_x p + \frac{\eta}{\rho}\Delta v \qquad\qquad \partial_x p = \eta\Delta v$

$\vec{e}_y$: $\quad 0 = -\frac{1}{\rho}\partial_y p \longrightarrow p = p(x,z)$

$\vec{e}_z$: $\quad 0 = -\frac{1}{\rho}\partial_z p - g \longrightarrow p = -\rho g z + f(x)$

$\Rightarrow \mathrm{div}\,\vec{v} = \partial_i v_i = \partial_x v = 0 \longrightarrow v = v(y,z)$

Symetria: v smere y sa nič nedeje $\Rightarrow$ $v = v(z)$, nie aj funkcia y

$\Rightarrow p = -\rho g z + f(x)$

$f'_x = \eta\,\partial_z\partial_z v \quad \Rightarrow$ Ľ.S. je funkcia x, P.S je funkcia z $\Rightarrow$ musia byť konštanty

$\Rightarrow f'_x = k \quad \Rightarrow f(x) = kx + q \qquad \Rightarrow p = -\rho g z + kx + q$

$\Rightarrow v''_{zz} = \frac{k}{\eta} \quad \Rightarrow v(z) = \frac{1}{2}\frac{k}{\eta}z^2 + k_1 z + k_2$

$\Rightarrow$ OP: $v(0) = 0 \quad \Rightarrow k_2 = 0$

$v(H) = V \quad \Rightarrow V = \frac{1}{2}\frac{k}{\eta}H^2 + k_1 H \longrightarrow k_1 = \frac{V}{H} - \frac{1}{2}\frac{k}{\eta}H$

$$\Rightarrow \vec{v}(x,y,z) = \vec{e}_z\left(\frac{1}{2}\frac{k}{\eta}z(z-H) + \frac{V}{H}z\right)$$

$$p(x,y,z) = -\rho g z + kx + q$$

$\Rightarrow$ pre $k = 0$ dostaneme $\quad \vec{v}(x,y,z) = \vec{e}_z\frac{Vz}{H}$, $\quad p(x,y,z) = -\rho g z + q \ \ldots$ } tlak aj rýchlosť závisí od výšky z

$\Rightarrow$ pre $k \neq 0$ máme $p(x,y,z) = -\rho g z + q + \underbrace{kx}$ → spád tlaku } to ovplyvní aj rýchlosť

$\vec{v}(x,y,z) = \vec{e}_z\left(\frac{1}{2}\frac{k}{\eta}z(z-H) + \frac{Vz}{H}\right)$

- metóda: variačný počet

↳ funkcionál $F[y]$ → hľadáme také y, že $F[y]$ je extremálne

↳ pre pohyb častíc = Newtonova mechanika $S[q]$ → účinok → rozmer J.s

$$S[q] = \int_{t_1}^{t_2} dt\, L(q, \dot{q}, t)$$

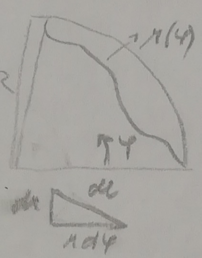
↳ rozmer energie = J

princíp najmenšieho účinku ⇐ variácia účinku → požiadavka na $\delta S = 0$ dáva Euler-Lagrangeove rovnice

↳ pre $F[y] = \int_{x_1}^{x_2} dx\, \mathcal{L}(y, y', x)$ → $\delta F = 0$ dáva rovnicu v rovnakom tvare ako E-L r., ale v iných premenných

$$q \longleftrightarrow y, \quad \dot{q} \longleftrightarrow y', \quad t \longleftrightarrow x$$

↓ môže byť ľubovoľná veličina ľubovoľného rozmeru (a nemusí mať ani správny rozmer)



$dl^2 = dr^2 + (r\,d\varphi)^2$

$$\frac{1}{2}mv^2 - \varkappa\frac{M(r)\,m}{r} = E \;\rightarrow\; v^2 = \frac{2}{m}\left(E + \varkappa\frac{m M(r)}{r}\right) \rightarrow v(R) = 0 = \frac{2}{m}E + \frac{2}{R}\varkappa M$$

$$E = -\varkappa\frac{Mm}{R}$$

$$M(r) = \int_0^r \rho\, 4\pi \xi^2\, d\xi = \rho 4\pi \frac{r^3}{3}$$

$$v^2 = 2\varkappa\left(\frac{M(r)}{r} - \frac{M}{R}\right) = 2\varkappa\left(\frac{4}{3}\pi r^2\rho - \frac{4}{3}\pi R^2\rho\right) = \frac{8}{3}\varkappa\pi\rho(r^2 - R^2)$$

$$v = \sqrt{\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)}$$

$$T = \int dt = \int \frac{dl}{v} = \int \frac{\sqrt{dr^2 + r^2 d\varphi^2}}{v} = \int_0^\alpha d\varphi \underbrace{\frac{\sqrt{r'^2 + r^2}}{\sqrt{\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)}}}_{\mathcal{L}}$$

$r(0) = R = r(\alpha)$

$$\mathcal{L}(r, r', \varphi) = \sqrt{\frac{r'^2 + r^2}{\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)}} \rightarrow r'\frac{\partial\mathcal{L}}{\partial r'} - \mathcal{L} = k$$

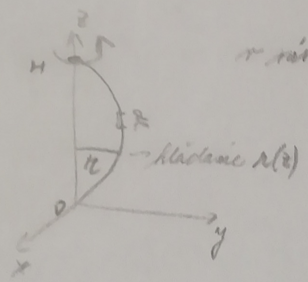
$$r'\frac{r'}{\sqrt{\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)}}\frac{1}{\sqrt{r'^2 + r^2}} - \sqrt{\frac{r'^2 + r^2}{\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)}} = k$$

$$\cancel{r'^2} - \cancel{r'^2} - r^2 = k\sqrt{\tfrac{8}{3}\varkappa\pi\rho(r^2 - R^2)}\sqrt{r'^2 + r^2} \rightarrow r^4 = k^2\frac{8}{3}\pi\rho\varkappa(r^2 - R^2)(r'^2 + r^2)$$

$$r'^2 = \frac{r^4}{k^2\frac{8}{3}\varkappa\pi\rho(r^2 - R^2)} - r^2 = \frac{r^4 - Cr^4 + Cr^2R^2}{\underbrace{k^2\tfrac{8}{3}\varkappa\pi\rho}_{C}(r^2 - R^2)} = \frac{r^4(1 - C) + r^2CR^2}{C(r^2 - R^2)}$$

$$\frac{dr}{d\varphi} = r\sqrt{\frac{r^2(1 - C) + CR^2}{C(r^2 - R^2)}} = r\sqrt{\frac{r^2 - C(r^2 - R^2)}{C(r^2 - R^2)}} \rightarrow \varphi + \beta = \int \frac{dr}{r}\sqrt{\frac{C(r^2 - R^2)}{r^2 - C(r^2 - R^2)}}$$

Švihadlo v rotácii



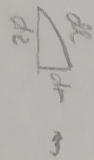
v sústave spojenej so švihadlom pôsobí odstredivá sila

$$\vec{F} = m\omega^2 r \frac{\vec{r}}{r}$$

$$\vec{F} = -\vec{\nabla} U \rightarrow U = -\frac{1}{2} m\omega^2 r^2$$

$$dU = dm\left(-\frac{1}{2}\right)\omega^2 r^2 = -\frac{1}{2}\omega^2 r^2 \rho\, dl = -\frac{1}{2}\omega^2 r^2 \rho \sqrt{1+r'^2}\, dz$$

$$U = \int_0^H \left(-\frac{1}{2}\right)\omega^2 \rho\, r^2 \sqrt{1+r'^2}\, dz \qquad L = \int_0^H \sqrt{1+r'^2}\, dz \qquad r(0) = 0,\ r(H) = 0$$



$$\rightarrow dl^2 = dz^2 + dr^2 \rightarrow dl = dz\sqrt{1+\frac{dr^2}{dz^2}}$$

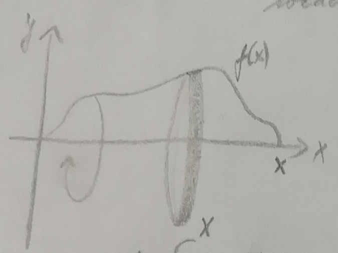
$$\mathcal{L} = -\frac{1}{2}\omega^2 \rho\, r^2 \sqrt{1+r'^2} + \lambda\sqrt{1+r'^2} = \mathcal{L}(z, r, r')$$

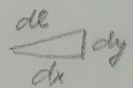
neobsahuje z $\Rightarrow$ $r'\frac{\partial \mathcal{L}}{\partial r'} - \mathcal{L} = \text{konšt.} = r'\left(-\frac{1}{2}\omega^2\rho r^2\right)\frac{2}{2}\frac{r'}{\sqrt{1+r'^2}} + \lambda\frac{r'}{\sqrt{1+r'^2}} +$

$$+ \frac{1}{2}\omega^2 \rho\, r^2 \sqrt{1+r'^2} - \lambda\sqrt{1+r'^2} \rightarrow r' = \ldots$$

$$\int dr = \int \ldots dz$$

daná plocha, maximálny objem → riešenie: sféra
↳ rotačný objekt





$$dl = \sqrt{dx^2 + dy^2} = dx\sqrt{1+y'^2}$$

$$dS = 2\pi y\, dl = 2\pi y\sqrt{1+y'^2}\, dx$$

$$dV = \pi y^2 dx$$

$$V = \int_0^X \pi y^2 dx \qquad S = \int_0^X 2\pi y\sqrt{1+y'^2}\, dx$$

$$\mathcal{L} = \pi y^2 + \lambda 2\pi y\sqrt{1+y'^2} \longrightarrow F[y] = \int_0^X \mathcal{L}(y, y'; x)\, dx = \int_0^X \left(\pi y^2 + \lambda 2\pi y\sqrt{1+y'^2}\right) dx$$

$$y'\frac{\partial \mathcal{L}}{\partial y'} - \mathcal{L} = C = y'\pi 2\lambda y\frac{y'}{\sqrt{1+y'^2}} - \pi y^2 - \lambda 2\pi y\sqrt{1+y'^2} = \frac{1}{\sqrt{1+y'^2}}\left[2\pi y\lambda y'^2 - 2\pi y\lambda(1+y'^2)\right] - \pi y^2$$

$$y(0) = 0 \rightarrow y = 0 \rightarrow C = 0$$

$$\Rightarrow \pi y^2 = \frac{-2\pi\lambda y}{\sqrt{1+y'^2}} \rightarrow y = \frac{-2\lambda}{\sqrt{1+y'^2}} \rightarrow y^2(1+y'^2) = 4\lambda^2 \rightarrow y'^2 = \frac{4\lambda^2}{y^2} - 1$$

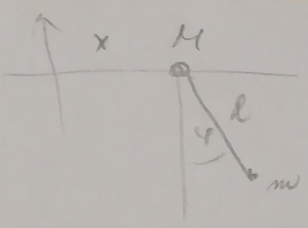
$$y' = \frac{1}{y}\sqrt{4\lambda^2 - y^2} \rightarrow dy\frac{y}{\sqrt{4\lambda^2 - y^2}} = dx \rightarrow x + k = \int 2\lambda\frac{d\xi\ \xi}{\sqrt{1-\xi^2}} = \left[\begin{array}{l}\sqrt{1-\xi^2} = t \\ \frac{-\xi}{\sqrt{1-\xi^2}}d\xi = dt\end{array}\right]$$

$$= -2\lambda\sqrt{1-\xi^2} = -2\lambda\sqrt{1-\frac{y^2}{4\lambda^2}} = -\sqrt{4\lambda^2 - y^2}$$

$$(x+k)^2 + y^2 = 4\lambda^2 \rightarrow \text{pre } x = 0 \rightarrow y = 0 \rightarrow k = \pm 2\lambda$$

$$(x+2\lambda)^2 + y^2 = 4\lambda^2 \rightarrow \text{pre } x = X \rightarrow y = 0 \rightarrow X^2 + 4\lambda X = 0 \rightarrow X = -4\lambda \rightarrow \lambda = -\frac{X}{4}$$

$$\left(x - \frac{X}{2}\right)^2 + y^2 = \frac{X^2}{4}$$



$$\frac{m}{M} = \varepsilon \ll 1$$

$$\vec{r}_m = (x + l\sin\varphi, -l\cos\varphi) \qquad \vec{r}_M = (x, 0)$$

$$\dot{\vec{r}}_m = (\dot x + l\dot\varphi\cos\varphi, l\dot\varphi\sin\varphi) \qquad \dot{\vec{r}}_M = (\dot x, 0)$$

$$T = \frac{1}{2}m\left[\dot x^2 + 2l\dot x\dot\varphi\cos\varphi + l^2\dot\varphi^2\right] + \frac{1}{2}M\dot x^2$$

$$U \equiv 0$$

1. $x \to \frac{\partial L}{\partial \dot x} = k_1 = (m+M)\dot x + ml\dot\varphi\cos\varphi = M\left[(1+\varepsilon)\dot x + \varepsilon l\cos\varphi\,\dot\varphi\right]$

2. $\frac{d}{dt}\frac{\partial L}{\partial\dot\varphi} - \frac{\partial L}{\partial\varphi} = 0 = ml\ddot x\cos\varphi - \cancel{ml\dot x\dot\varphi\sin\varphi} + ml^2\ddot\varphi + \cancel{ml\dot x\dot\varphi\sin\varphi} =$
$= ml\left[\ddot x\cos\varphi + l\ddot\varphi\right] \to \ddot x\cos\varphi + l\ddot\varphi = 0$

3. $x(0) = 0$, $\dot x(0) = 0$ P.P. $\varphi(0) = 0$, $\dot\varphi(0) = \Omega$ $\to k_1 = M\varepsilon l\Omega \to \varepsilon l\Omega = (1+\varepsilon)\dot x + \varepsilon l\cos\varphi\,\dot\varphi$

4. $x(t) = x_0(t) + \varepsilon x_1(t)$
$\varphi(t) = \varphi_0(t) + \varepsilon\varphi_1(t)$ $\to \sim\varepsilon^2$ zanedbáme

5. $(\ddot x_0 + \varepsilon\ddot x_1)\cos(\varphi_0 + \varepsilon\varphi_1) + l(\ddot\varphi_0 + \varepsilon\ddot\varphi_1) = 0$
$\varepsilon l\Omega = (1+\varepsilon)(\dot x_0 + \varepsilon\dot x_1) + \varepsilon l\cos(\varphi_0 + \varepsilon\varphi_1)(\dot\varphi_0 + \varepsilon\dot\varphi_1)$
$\to \cos(\varphi_0 + \varepsilon\varphi_1) = \cos\varphi_0 + \varepsilon\frac{d}{d\varepsilon}\left(\cos(\varphi_0 + \varepsilon\varphi_1)\right)\Big|_{\varepsilon=0} = \cos\varphi_0 - \varepsilon\sin\varphi_0\cdot\varphi_1$

ε^0: $\ddot x_0\cos\varphi_0 + l\ddot\varphi_0 = 0 \to \ddot\varphi_0 = 0 \Rightarrow \dot\varphi_0 = \text{konšt.} \overset{P.P.}{=} \Omega \to \varphi_0 = \Omega t$
$0 = \dot x_0 \xrightarrow{P.P.} x_0 = 0$

ε^1: $-\ddot x_0\varphi_1\sin\varphi_0 + \ddot x_1\cos\varphi_0 + l\ddot\varphi_1 = 0 \xrightarrow{x_0,\,\varphi_0} 0 + \ddot x_1\cos(\Omega t) + l\ddot\varphi_1 = 0$
$l\Omega = \dot x_1 + \dot x_0 + l\cos\varphi_0\cdot\dot\varphi_0$ $\qquad l\Omega = \dot x_1 + 0 + l\Omega\cos(\Omega t)$
$\dot x_1 = l\Omega(1 - \cos(\Omega t)) \to x_1(t) = l\Omega t - l\sin(\Omega t) + C \xrightarrow{P.P.} C = 0$
$l\Omega^2\sin(\Omega t)\cos(\Omega t) + l\ddot\varphi_1 = 0 \to \ddot\varphi_1 = -\Omega^2\sin(\Omega t)\cos(\Omega t) = -\frac{\Omega^2}{2}\sin(2\Omega t)$
$\dot\varphi_1 = \frac{\Omega^2}{2}\frac{1}{2\Omega}\cos(2\Omega t) + C_1$ $\qquad$ P.P. $\dot\varphi_1 = 0$ (lebo $\dot\varphi_0 = \Omega = \dot\varphi$)
$\varphi_1 = \frac{\Omega}{4}\frac{1}{2\Omega}\sin(2\Omega t) + C_1 t + C_2 \xrightarrow{P.P.} 0 = \frac{\Omega}{4} + C_1 \to C_1 = -\frac{\Omega}{4}$
$C_2 = 0$
$\varphi_1 = \frac{1}{8}\sin(2\Omega t) - \frac{\Omega}{4}t$

$\Rightarrow x(t) = 0 + \frac{m}{M}\left[l\Omega t - l\sin(\Omega t)\right]$
$\varphi(t) = \Omega t + \frac{m}{M}\left[\frac{1}{8}\sin(2\Omega t) - \frac{\Omega}{4}t\right]$