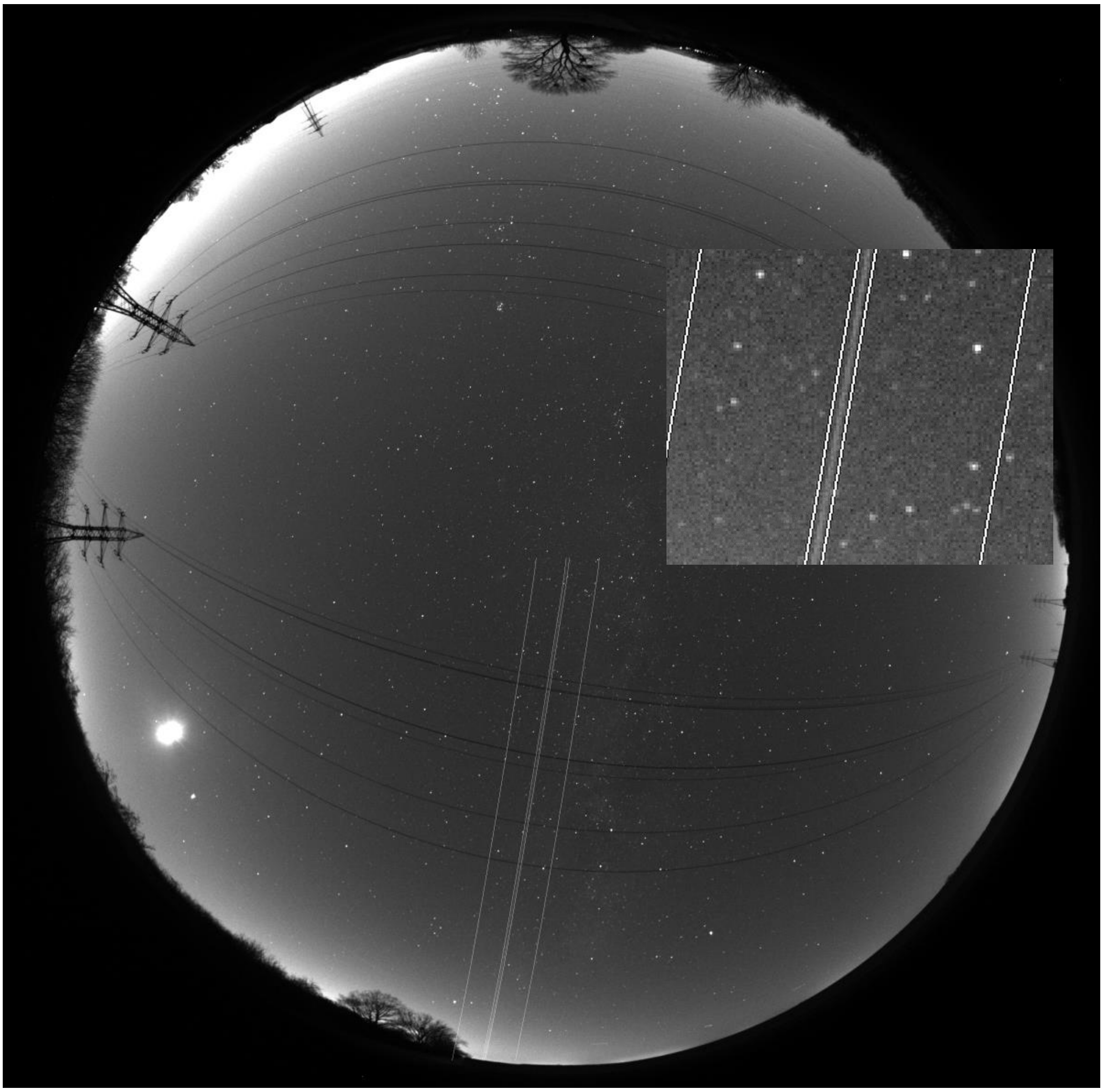
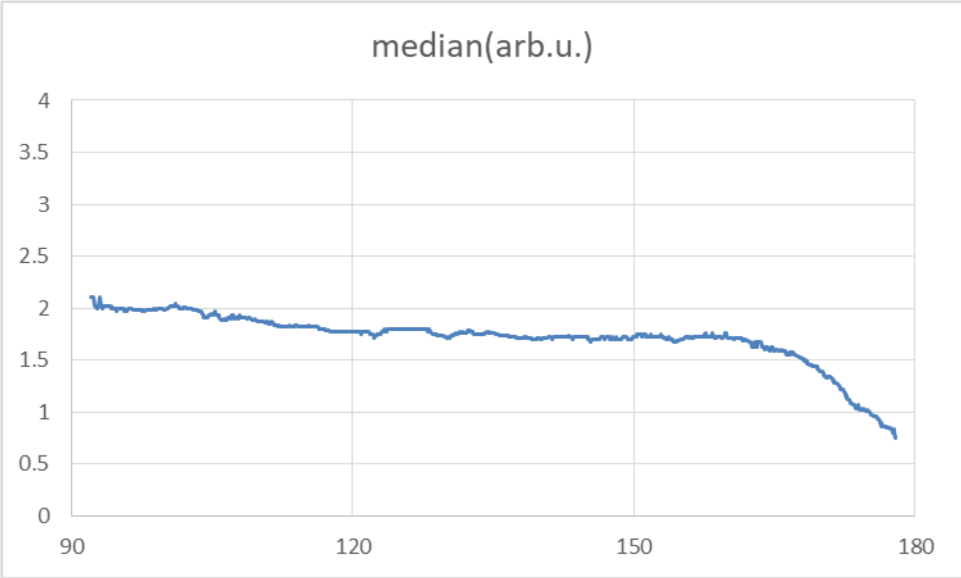
Procedure for finding the vertical profile of aerosols

1. Input data

The input is the height profile of the laser beam obtained by the "green_laser" program - as in the measurement of the scattering phase function. However, when measuring the elevation profile, the laser is much further, the signal is much more noisy, so the extracted data must be filtered by the median to a greater extent. A median filter with a width of 150 points proved to be optimal in terms of minimal noise and sufficient resolution.



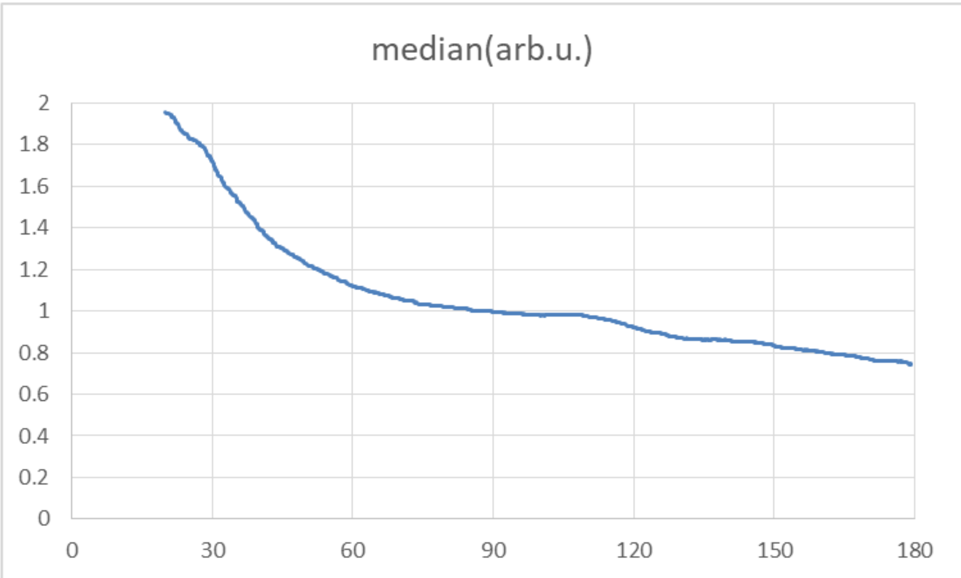
An image of the laser beam before reading-out the background. The beam area and the background level measurement area are highlighted. Inset: Detail of the laser beam.



Vertical profile of the laser beam (median-filtered 150 points)

In addition to the vertical profile, we also need a phase scattering function, obtained by aiming the laser above the camera at low elevation angle (20 degrees in our case). Only its part from 90 degrees up to about 175 degrees will be used.

The physical input of the elevation profile and the scattering phase function are the "green_laser_data.txt" files, which are the output of the "green_laser" program.

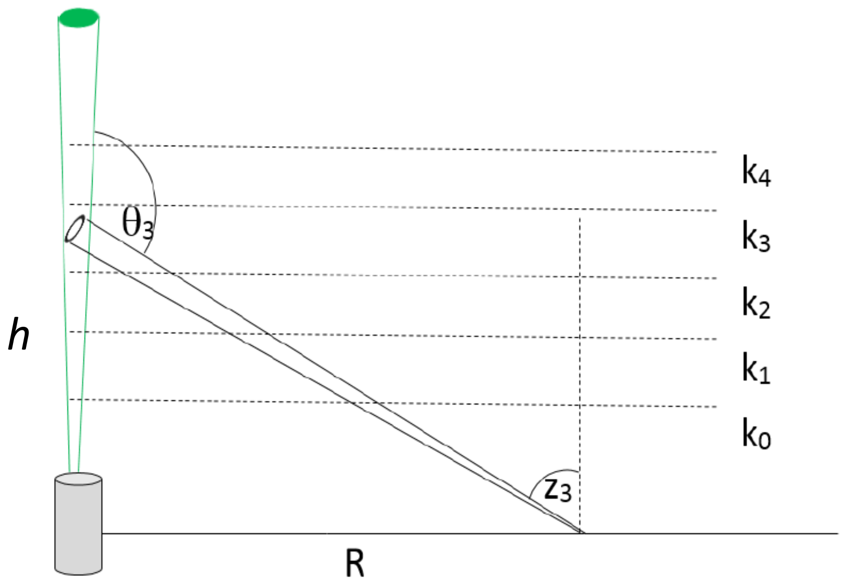


Normalised scattering phase function determined from a beam with an elevation of 20 degrees

2. Solution diagram

The vertical profile of the laser beam was measured with a high resolution of approximately 0.1 ° (1000 points in the range of angles 0 ° - 90 °). The first step is to reduce the resolution with a constant step at the zenith angle (**typically 0.5 ° -1 °**), which leads to further noise reduction. We obtain height layers with unequal thickness: Near the surface, where the signal is strong, the height resolution is high, at higher altitudes the height resolution is lower.

At the same time, we set the minimum zenith angle (ie the maximum height hmax) for which we process the signal at typically **10 ° -7 °** (about 3,000 m - 5,000 m), because for smaller zenith angles, the error in determining the height increases greatly.



Definition of used symbols

The recorded signal is proportional to the power of the radiation incident on the camera and the exposure time. Since we are processing only one image of the laser beam, we can write:

$$W_{sca,\lambda}(z) \propto k_{ext,\lambda}(z) \exp\left\{-\left(\frac{1+\cos z}{\cos z}\right)\int_0^{\frac{R}{\tan z}} k_{ext,\lambda}(\xi)d\xi\right\}\frac{P_\lambda(\theta)}{4\pi}\Delta z$$

The zenite angle *z* and the height *h* are related as:

$$h(z) = \frac{R}{\tan z}$$

Using this relationship and taking into account that the pixel size (detector field of view) is constant (images are corrected for vigneting), we can write for the vertical profile of the recorded signal:

$$\frac{W_{sca,\lambda}(h)}{P_\lambda(\theta(h))} \propto k_{ext,\lambda}(h) \exp\left\{-\left(\frac{1}{\cos z(h)}+1\right)\int_0^{h} k_{ext,\lambda}(\xi)d\xi\right\}$$

It is obvious that we can extract from the measured signal only a quantity proportional to $k_{ext,\lambda}(h)$. Therefore, we need to standardize the calculated values:

$$\int_0^{h_{max}} k_{ext,\lambda}(h) = AOD(h_{\max}) + ROD(h_{\max})$$

where $AOD(h_{max})$ is the aerosol optical thickness between the surface and the maximum height to be processed h_{max} a $ROD(h_{max})$ is the optical thickness of the molecular atmosphere between the surface and the maximum height processed h_{max}.

For the wavelength of 532 nm applies:

$$ROD(h_{\max}) = 0.116\left(1 - \exp\left(-\frac{h_{max}}{8000\ \mathrm{m}}\right)\right)$$

Since most aerosols are lower than h_{max}, $AOD(h_{max})$ will usually differ little from AOD. In addition, we will show below that the choice of AOD does not have a critical effect on the determination of the relative altitude profile of aerosols..

We will solve the above integral equation iteratively from the relation:

$$k_{ext,\lambda}(h) \propto \frac{W_{sca,\lambda}(h)}{P_\lambda(\theta(h))} \exp\left\{\left(\frac{1}{\cos z(h)} + 1\right)\int_0^h k_{ext,\lambda}(\xi)d\xi\right\}$$

Procedure:

1. As a zero estimate, we neglect the damping term (the integral is equal to zero), that is $k_{ext,\lambda}^{(0)} = 0$.
2. Calculate values proportional to $k_{ext,\lambda}^{(1)}$. After their normalization to $AOD(h_{\max}) + ROD(h_{\max})$ we get the first estimate $k_{ext,\lambda}^{(1)}$.
3. Find the maximum relative change in this step
$$max\left\{\frac{\left|k_{ext,\lambda}^{(1)}(h) - k_{ext,\lambda}^{(0)}(h)\right|}{k_{ext,\lambda}^{(0)}(h)}\right\}$$
4. Repeat points 2 and 3 until the change is negligible.

The method converges very quickly, usually just a few steps to achieve a relative change of less than 10^{-6}.

After calculating $k_{ext,\lambda}(h)$ we can determine the part belonging to the aerosol:

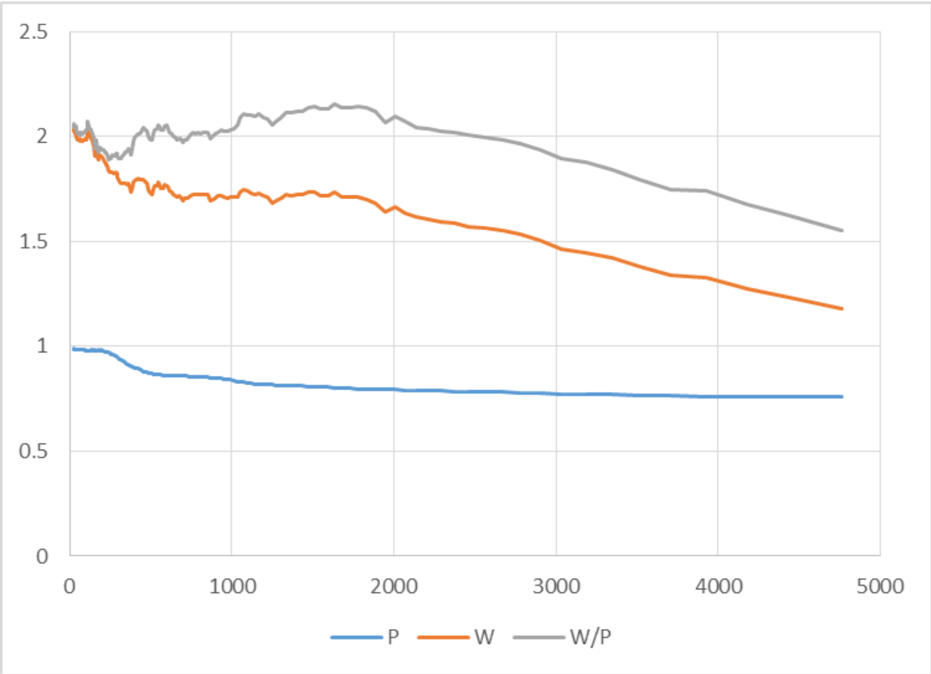
$$k_{A,ext,\lambda}(h) = k_{ext,\lambda}(h) - k_{R,ext,\lambda}(h)$$

where

$$k_{R,ext,\lambda}(h) = \frac{0.116}{8000\ \mathrm{m}} \exp\left(-\frac{h}{8000\ \mathrm{m}}\right)$$

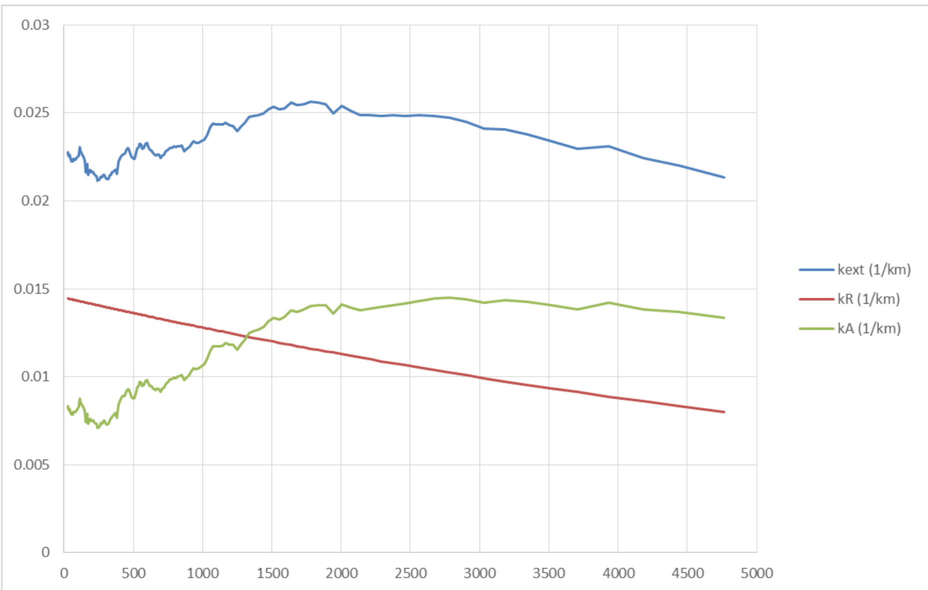
3. Results

The recorded signal *W*, the scattering phase function *P* and their ratio W / *P* as a function of height *h* are in a reduced resolution of **0.5 °** and with the applied limit of the zenith angle of **7 °** are shown in the following figure:



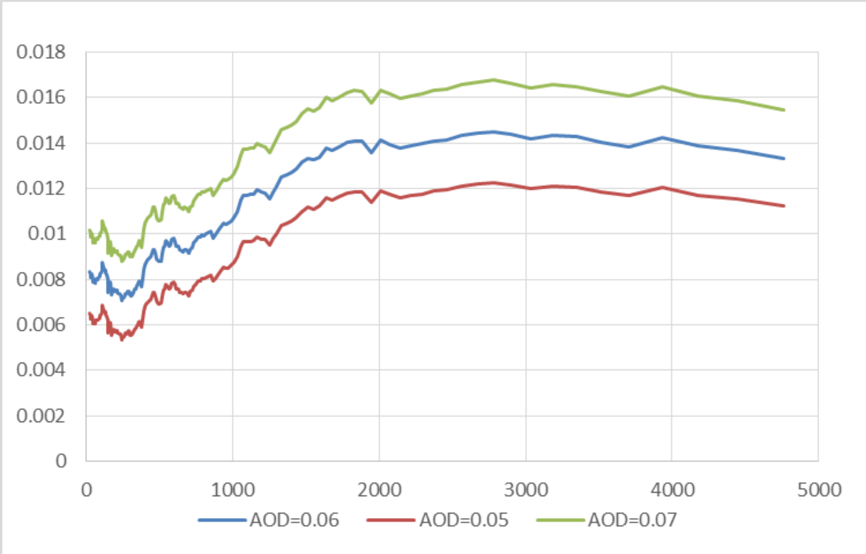
The recorded signal W, the scattering phase function P and their ratio W/P as a function of height

Vertical profile of *k*, k_A and k_R for AOD = 0.06 is in the following figure:



Vertical profile of the total extinction coefficient k_{ext} and its components for the molecular atmosphere k_R and aerosol k_A

Next, we examined the effect of choosing the AOD value on the vertical profile. We repeated the calculation for AOD = 0.05, 0.06 and 0.07, i.e. in the range of +/- 17% around the value 0.06. Of course, the choice of AOD has a great influence on the absolute values of the extinction coefficient k_A:



Vertical profile of aerosol extinction coefficient k_A *for various AOD*

However, if we want to obtain only the altitude profile and we standardize the values to 1000 m, for example, the variance is significantly lower:

Vertical profile of aerosol extinction coefficient k_A *(standardized to a height of 1000 m) for various AOD*

Obviously, the choice of AOD (and thus the choice of beam attenuation size) will have the greatest effect at high altitudes, when the beam path is longest. Together with the fact that the height determination is inaccurate at small zeniths, it predetermines the method for obtaining a height profile only up to a height equal to several times the distance of the camera from the laser, i.e. typically up to 2-3 km (for a 1 W laser).

4. Decription of the program „vertical_profile“

The program calculates the vertical profile k_{ext}, k_A a k_R from the extracted laser beam image data (extraction is done with the "green_laser" program and was described in the section dedicated to scattering phase function). The program reads the parameters from the file "vertical_profile.cfg", whose typical content is:

```
green_laser_data_600m_90deg_new.txt

green_laser_data_150m_20deg_new.txt

7
0.5

0.06

1 = filename - vertical extracted data
2 = filename - overhead extracted data

3 = minimum zenith angle
4 = vertical resolution in degrees

5 = aerosol optical depth
```

The first two parameters are the names of the files with the extracted data from the photos. The first is a profile for a laser aimed vertically, the second for a laser aimed overhead the camera. The third parameter is the maximum zenith angle for which we still process the data (determines the maximum height). The fourth parameter is the resolution of the processed data in degrees. The last parameter is $AOD(h_{max})$.

Typical screen-output of the program:

```
Program 'vertical_profile' calculates vertical profile of the extinction
coefficient.
Configuration data are read from 'vertical_profile.cfg'-file.
Reading vertical profile...
953 points read.
Reading scattering angular dependence...
1768 points read.
Correcting and averaging vertical profile, step = 0.5 deg
166 points calculated - up to 4763.42 m
AOD = 0.06      RayleighOD = 0.052046
Optimizing kext...
Step 1   Error: 0.107366
Step 2   Error: 0.00359542
Step 3   Error: 8.05445e-005
Step 4   Error: 3.19339e-006
Step 5   Error: 3.54821e-007
OK
Writing 'kext_vprofil.txt' file...
OK
Press any key to continue . . .
```

Output-file 'kext_vprofil.txt' has following structure:

```
# Height (m)     kext (1/km)    kR (1/km)      kA (1/km)
26.5616          0.0227913      0.0144519      0.0083394
29.9212          0.0227015      0.0144459      0.0082556
34.9632          0.0225292      0.0144368      0.00809243
40.5601          0.02264        0.0144267      0.00821338
45.6065          0.0223886      0.0144176      0.00797105
50.6327          0.0222689      0.0144085      0.00786035
56.2289          0.0223529      0.0143984      0.00795445
...
```