METÓDY RIEŠENIA FYZIKÁLNYCH ÚLOH 2 zima26 – Príklady 3

VZOROVÉ RIEŠENIA

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Príklad 1

$$L = \frac{m}{2}\left[\dot{x}^2 + \left(\frac{l}{2}\right)^2\cos^2\theta\,\dot{\theta}^2\right] + \frac{ml^2}{24}\dot{\theta}^2 - \frac{mgl}{2}\sin\theta$$

$$\dot{x} = \text{const.} = 0 \quad , \quad x = 0$$

$$H = \left(\frac{m}{2}\frac{l^2}{4}\cos^2\theta + \frac{ml^2}{24}\right)\dot{\theta}^2 + \frac{mgl}{2}\sin\theta = \frac{mgl}{2}\sin\theta_0$$

$$\left(\frac{d\theta}{dt}\right)^2 = \frac{(8/l)\frac{1}{2}(\sin\theta_0 - \sin\theta)}{\frac{1}{8}\cos^2\theta + \frac{1}{24}}$$

$$\sqrt{8/l}\,t = \int_0^{\theta_0} \sqrt{\frac{\frac{1}{4}\cos^2\theta + \frac{1}{2}}{\sin\theta_0 - \sin\theta}}\,d\theta$$

$$\Delta x_{end} = \frac{l}{2} - \frac{l}{2}\cos\theta_0$$

Príklad 2

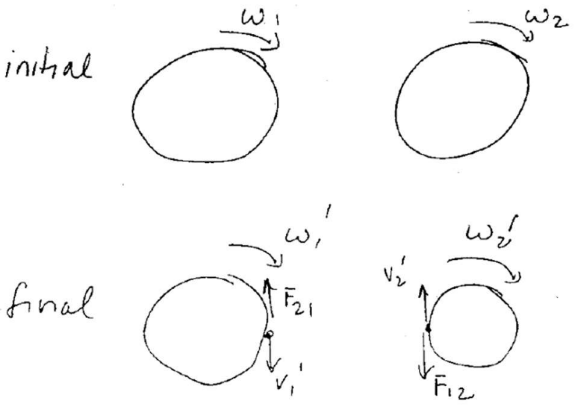
- Introduce the tension T in the rope and the force F which the mass M_h exerts to the right on the sphere. Use X, Y and x for the laboratory coordinates of M_h, M_v and m respectively.
- Write down equations for the acceleration of the cm of each mass:

$$\begin{aligned} M_h\ddot{X} &= T - F \\ m\ddot{a} &= F \\ M_v\ddot{Y} &= T - M_v g \end{aligned}$$

- Let θ represent the angular orientation of the sphere (increasing with clockwise motion) and write the equation for the angular acceleration of the sphere and the relation between $\ddot{\theta}$, $\ddot{a}$ and $\ddot{X}$:

$$\begin{aligned} \frac{2}{5}mR^2\ddot{\theta} &= -FR \\ \ddot{a} &= \ddot{X} + R\ddot{\theta} \end{aligned}$$

- These five equations can then be solved for T, F, $\ddot{\theta}$, $\ddot{X}$, $\ddot{Y}$,



Since the tangential final speeds must be the same (but opposite directions)

$$\omega_1' r_1 = -\omega_2' r_2 \quad (4)$$

$$\Rightarrow \omega_2' = -\frac{r_1}{r_2}\omega_1'$$

Since the integral of the torque equals the change in angular momentum

$$I_1(\omega_1' - \omega_1) = -\int |r_1||F_{21}|dt \quad (5)$$

and $I_2(\omega_2' - \omega_2) = -\int |r_2||F_{12}|dt$ $\rightarrow$ but $|F_{21}| = |F_{12}|$ from Newtons 3rd law

$$\therefore \quad \frac{I_1}{r_1}(\omega_1' - \omega_1) = \frac{I_2}{r_2}(\omega_2' - \omega_2)$$

Subst in for $\omega_2' = -\frac{r_1}{r_2}\omega_1'$

$$\therefore \quad \frac{I_1}{r_1}\omega_1' + \frac{I_2}{r_2}\cdot\frac{r_1}{r_2}\omega_1' = \frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2$$

$$\omega_1' = \frac{\frac{I_1}{r_1}\omega_1 - \frac{I_2}{r_2}\omega_2}{\frac{I_1}{r_1} + I_2\frac{r_1}{r_2}} = \frac{\frac{L_1}{r_1} - \frac{L_2}{r_2}}{\frac{I_1}{r_1} + I_2\frac{r_1}{r_2}}$$

with $I = \frac{1}{2}mr^2$ and some algebra (2)

$$\boxed{\omega_1' = \frac{m_1 r_1 \omega_1 - m_2 r_2 \omega_2}{(m_1 + m_2) r_1}} \quad (3)$$

Príklad 3

Answer:

$$C = \frac{dQ}{dT} = \frac{dU_{gas}}{dT} + \frac{dU_{spring}}{dT}$$

- the heat supplied to the system goes into energy of the gas and energy of the spring (gas does work on the spring, but this work is stored in spring inside the system, so we don't have to include it twice; one can also think about the entire system doing no work on its surroundings and only the internal energy of the system changing, which includes energy of the gas and spring).

Force of the spring compressed by x balances pressure

$$pS = kx \qquad pV = pSx = kx^2 = \nu RT$$

and using this connection between x and T we get

$$C = \frac{3}{2}\nu R + kx\frac{dx}{dT} = \frac{3}{2}\nu R + kx\frac{1}{2kx}\nu R = 2\nu R \qquad \boxed{C = 2\frac{p_0 V_0}{T_0}}$$